Corso di Laurea in Scienze Naturali
Corso di Laurea in Scienze Geologiche

TRIGONOMETRIA

Docente: Marianna Saba

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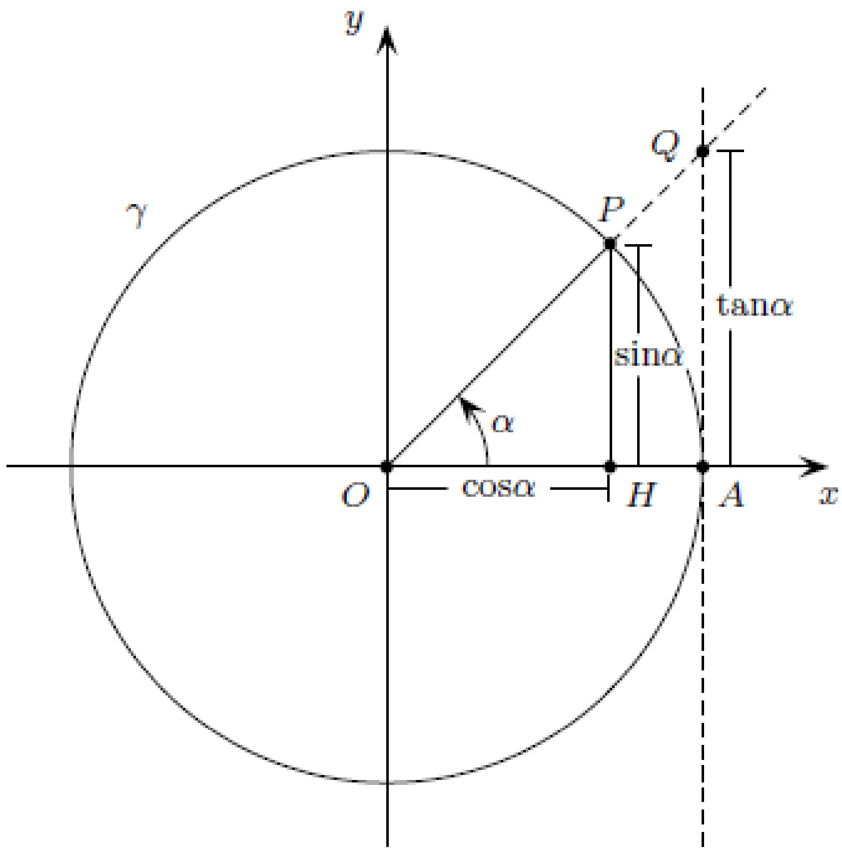
Circonferenza goniometrica



- A=(1,0)
- $\alpha = \frac{arco\ AP}{AO}$ radianti

$\frac{\pi}{2}\ rad = 90°;\ \ \pi\ rad = 180°;$

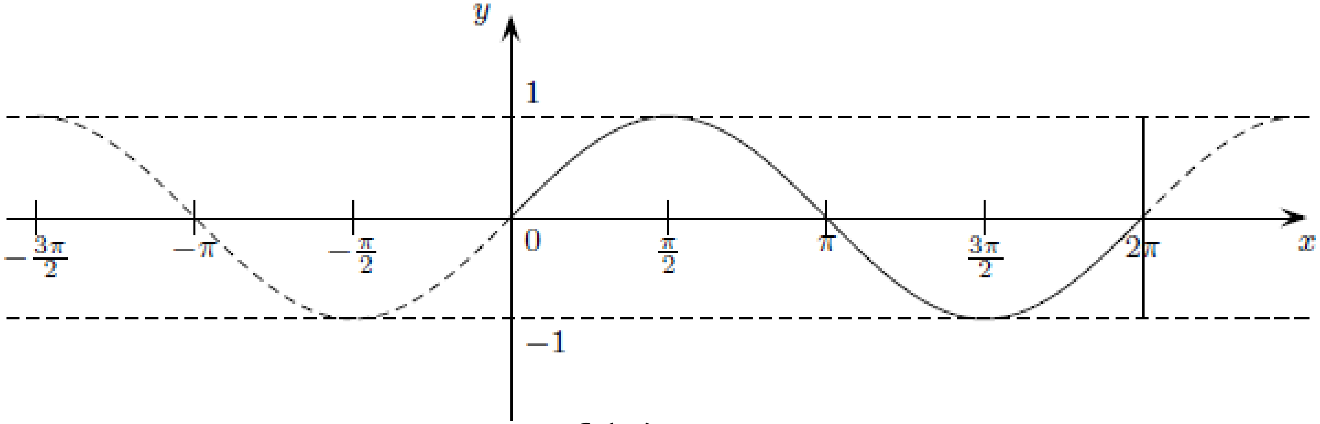
$\frac{3\pi}{2}\ rad = 270°;\ 2\pi\ rad = 360°$

- α positivo quando P si muove in senso antiorario.
- $\forall\alpha \in [0, 2\pi]$ definiamo

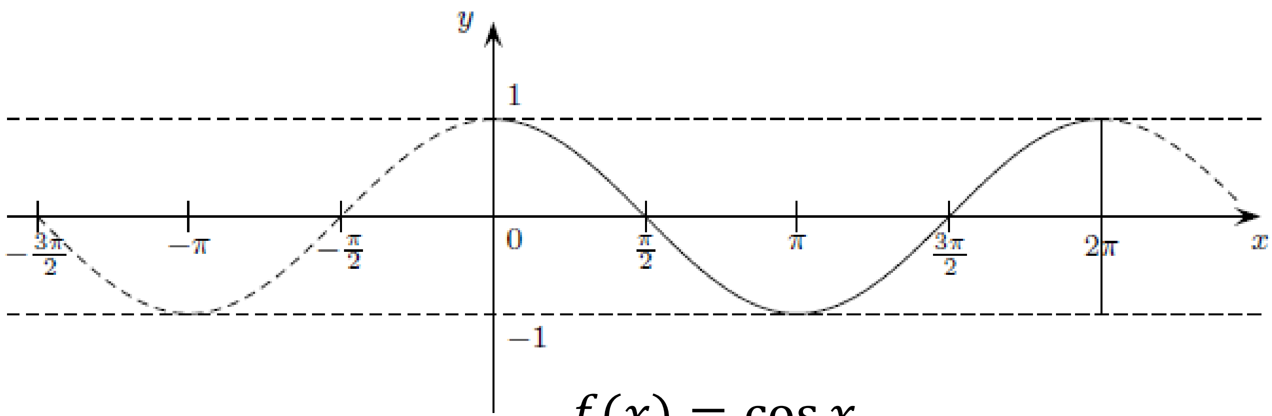
  $\sin\alpha$ =ordinata di P

  $\cos\alpha$= ascissa di P

Grafici delle funzioni $\sin x$ e $\cos x$



$f(x) = \sin x$



$f(x) = \cos x$

Proprietà della funzione $\sin x$

- **Funzione periodica** di **periodo** $\mathbf{T = 2\pi}$:
  $\sin x = \sin(x + T), \forall x \in R$

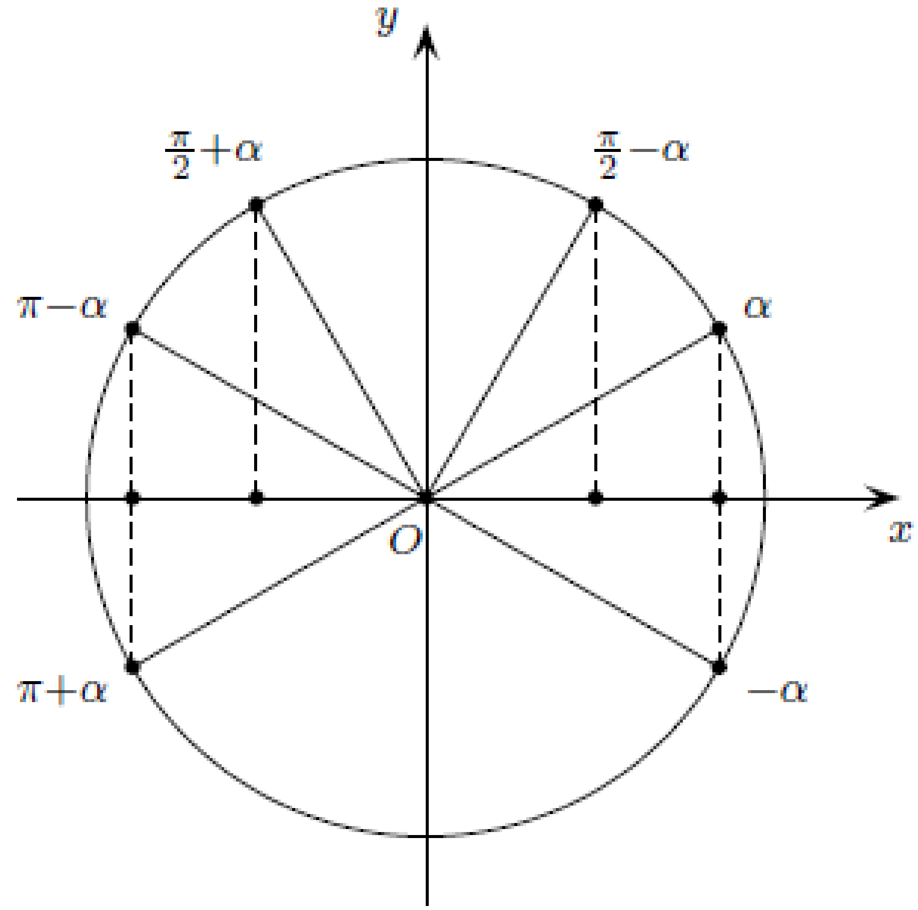
in generale:

$\sin x = \sin(x + kT), k \in Z$

- $\sin(-x) = -\sin x$
  (→ $\sin x$ è funzione dispari)

- $\sin(x + \pi) = -\sin x$

- $\sin(\pi - x) = \sin x$



Proprietà della funzione $\cos x$

- **Funzione periodica** di **periodo** $\mathbf{T = 2\pi}$:

  $\cos x = \cos(x + T)\,, \forall x \in R$

in generale:

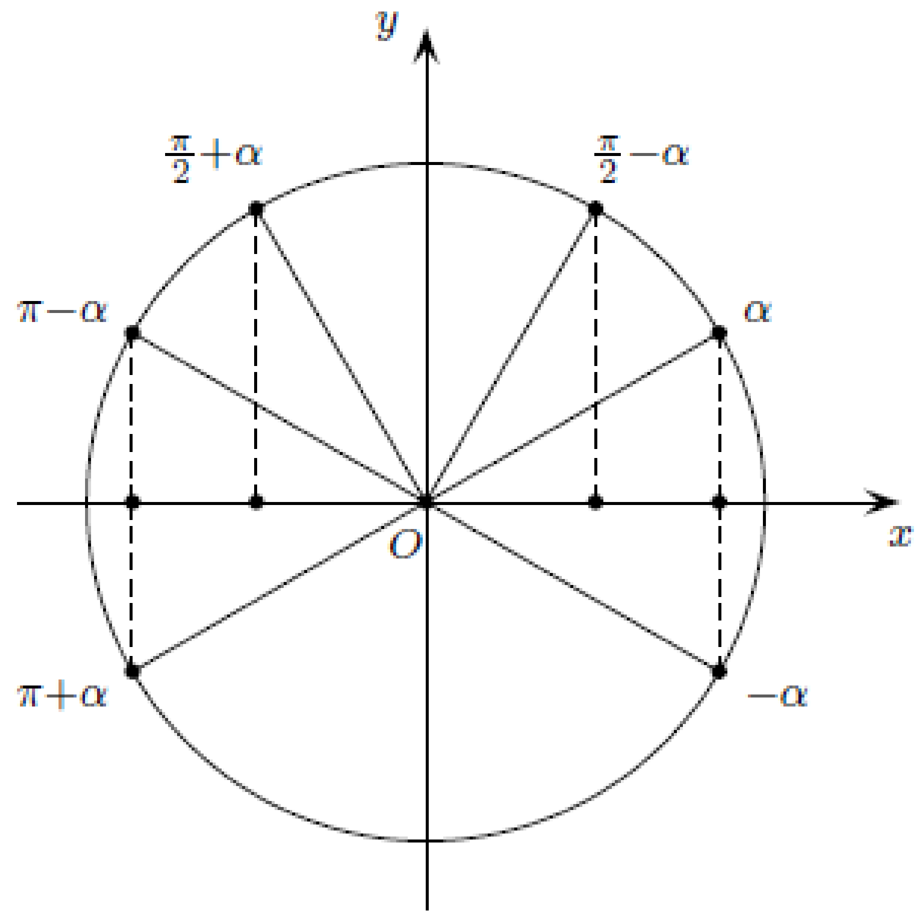
$\cos x = cos(x + kT), k \in Z$

- $\cos x = \cos(-x)$

  ($\rightarrow$ $\cos x$ è funzione pari)

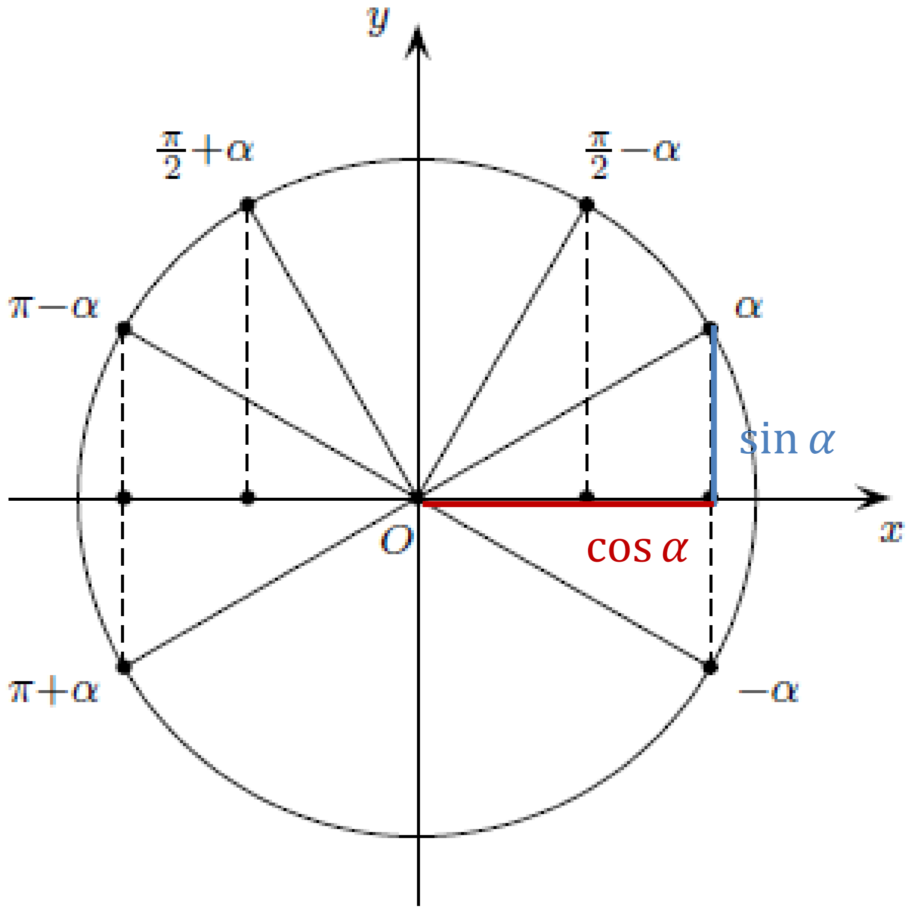
- $\cos(x + \pi) = -\cos x$

- $\cos(\pi - x) = -\cos x$



Ulteriori proprietà

- $\sin\left(x + \frac{\pi}{2}\right) = \cos x$
- $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$



Relazione fondamentale:

$$\boldsymbol{\sin^2 x + \cos^2 x = 1}$$

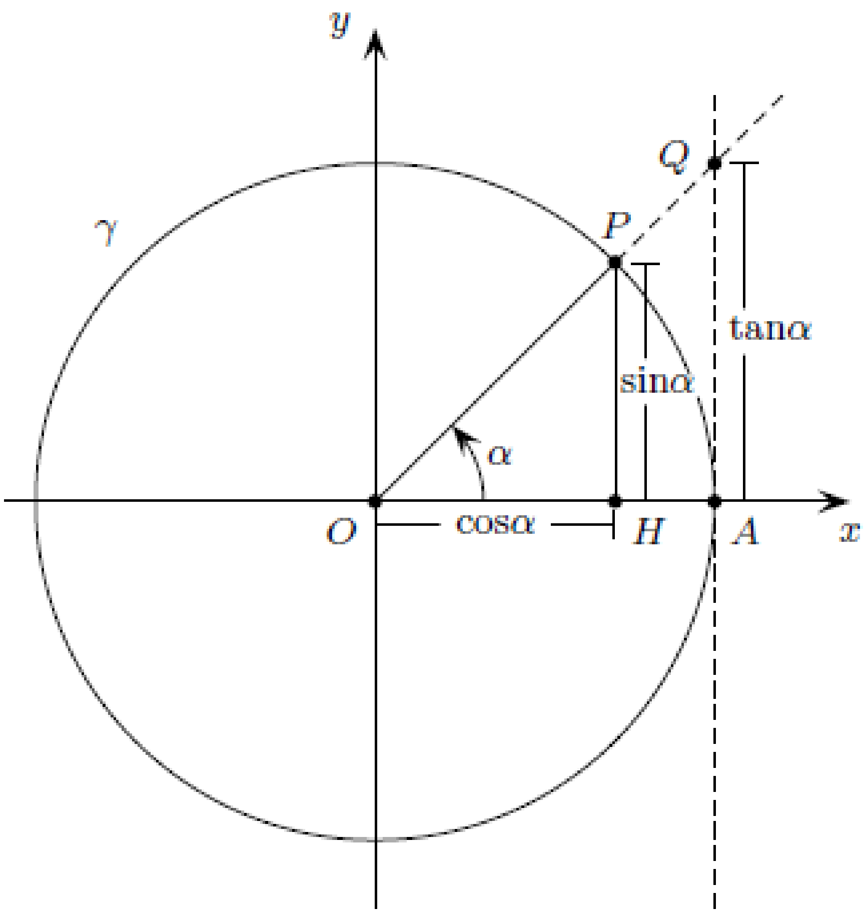
(dal Teorema di Pitagora)

Formule di somma, sottrazione, duplicazione e bisezione per $\sin\alpha$ e $\cos\alpha$

- $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$
  $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
- $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

La funzione $\tan x$



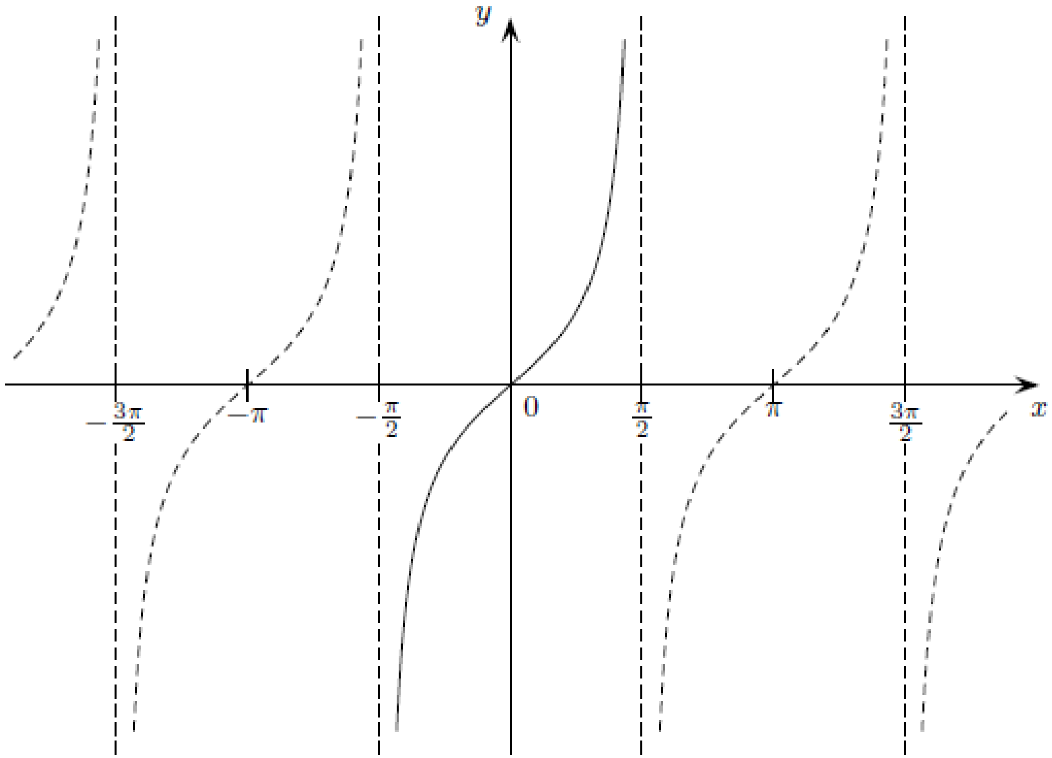
- $\tan x = \dfrac{\sin x}{\cos x}$

$\forall x \in R\colon\ \cos x \neq 0$, cioè
$\forall x \in R\colon x \neq \dfrac{\pi}{2} + k\pi,\ \ k \in Z$

- Funzione periodica di periodo $\boldsymbol{T = \pi}$

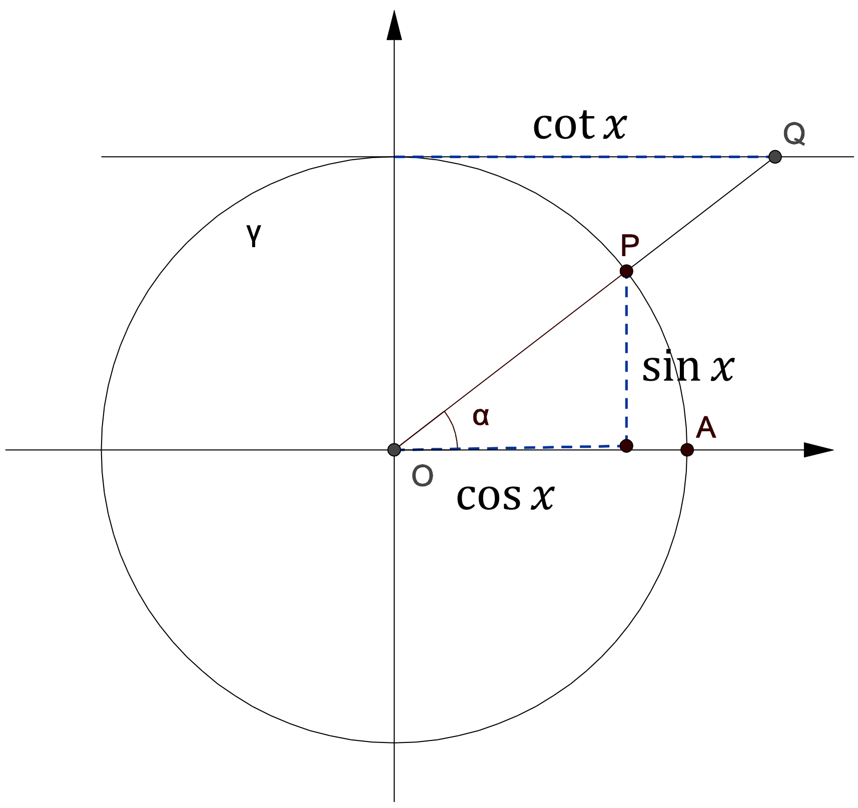
$\tan x = \tan(x + k\pi)$, $k \in Z$

Grafico della funzione $\tan x$



$$f(x) = \tan x$$

La funzione $\cot x$



- $$\cot x = \frac{\cos x}{\sin x}$$

  $\forall x \in R\colon \ \sin x \neq 0$, cioè

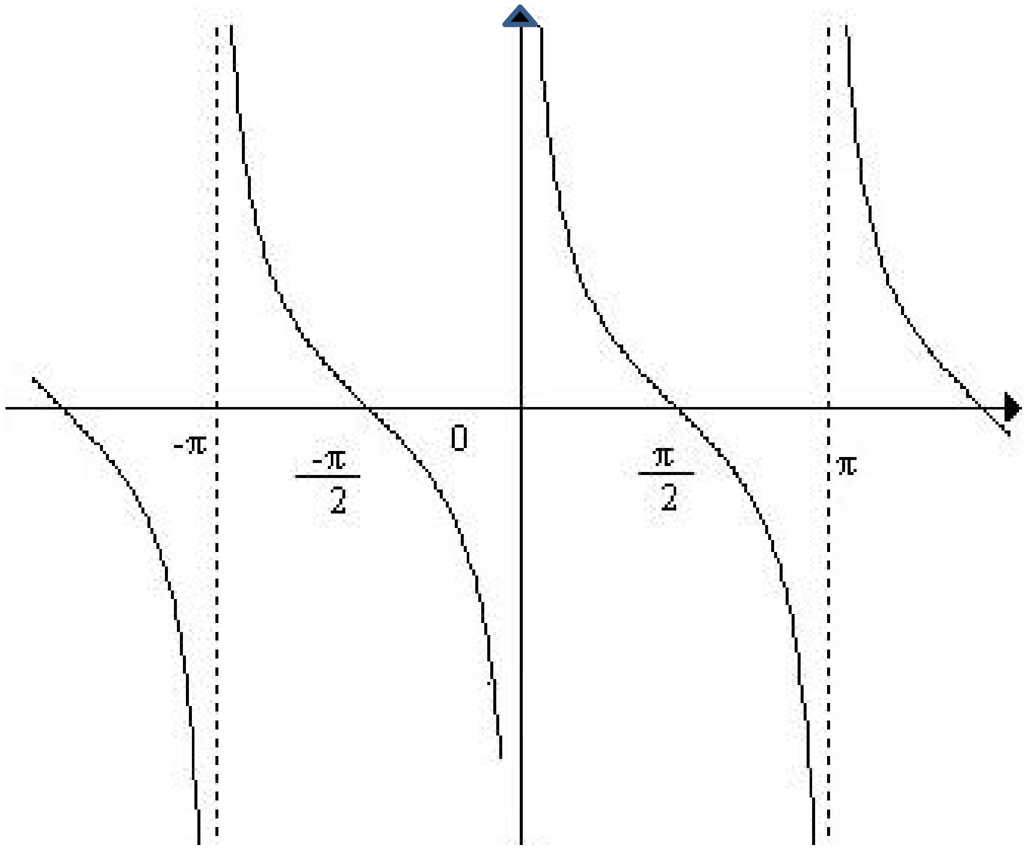
  $\forall x \in R, x \neq k\pi, \ \ k \in Z$

- Funzione periodica di periodo $\boldsymbol{T = \pi}$

  $\cot x = \cot(x + k\pi)$, $k \in Z$

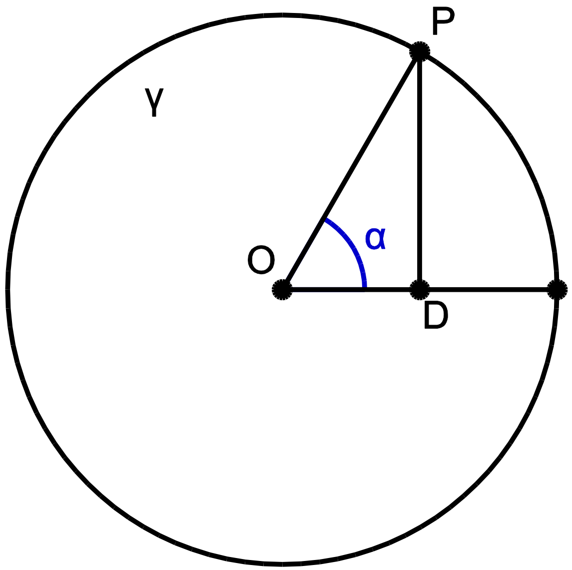
- $$\cot x = \frac{1}{\tan x}$$

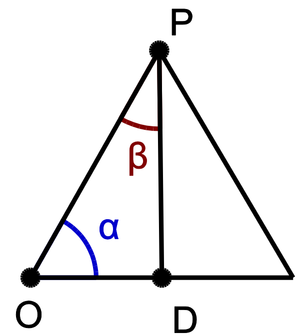
Grafico della funzione $\cot x$



f(x)= $\cot x$

Dimostrazione: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$





$\alpha = 60° = \frac{\pi}{3}$ $\beta = 30° = \frac{\pi}{6}$

$OP = 1$ OD= $\frac{1}{2}$

$$PD = \sin\alpha = \sin\frac{\pi}{3} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

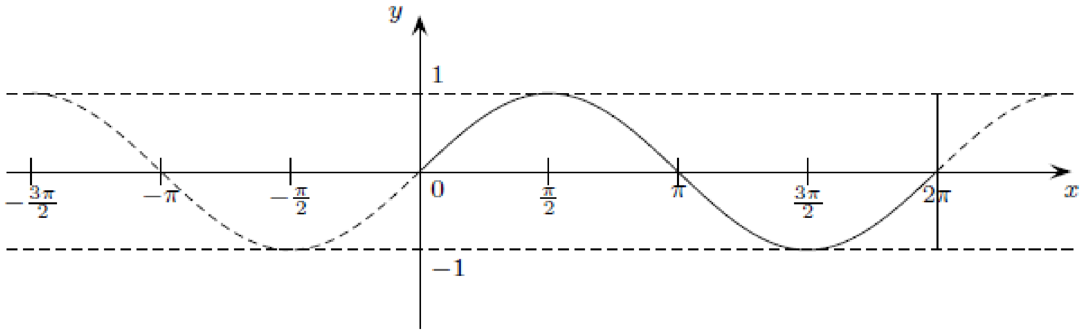
Angoli notevoli

- $\alpha = \frac{\pi}{4}:$ $\sin\alpha = \frac{\sqrt{2}}{2}; \cos\alpha = \frac{\sqrt{2}}{2}$
- $\alpha = \frac{\pi}{3}:$ $\sin\alpha = \frac{\sqrt{3}}{2}; \cos\alpha = \frac{1}{2}$
- $\alpha = \frac{\pi}{6}:$ $\sin\alpha = \frac{1}{2}; \cos\alpha = \frac{\sqrt{3}}{2}$

Calcolare $\tan\frac{\pi}{6}$, $\cot\frac{\pi}{3}$, $\tan\frac{\pi}{4}$

Funzioni inverse: $f^{-1}(x) = \arcsin x$

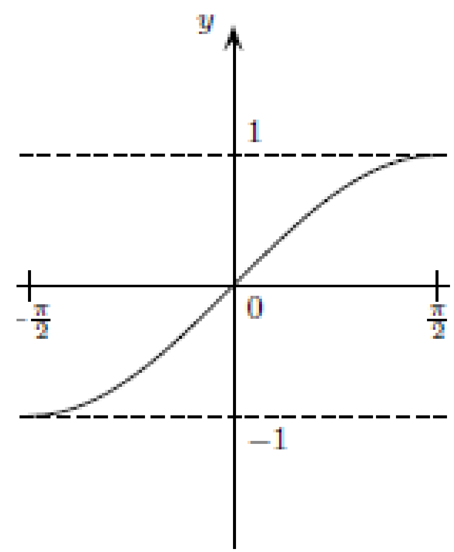
- $f: R \to R$ definita da $f(x) = \sin x$



NON è invertibile

- E' invertibile se definisco

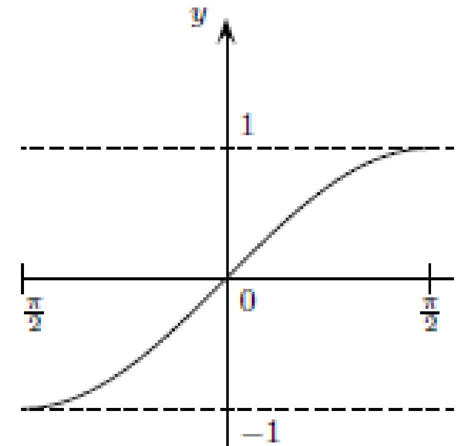
$$f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$$
$$f(x) = \sin x$$



La funzione arcsin(x)

$$f\colon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to[-1,1]$$

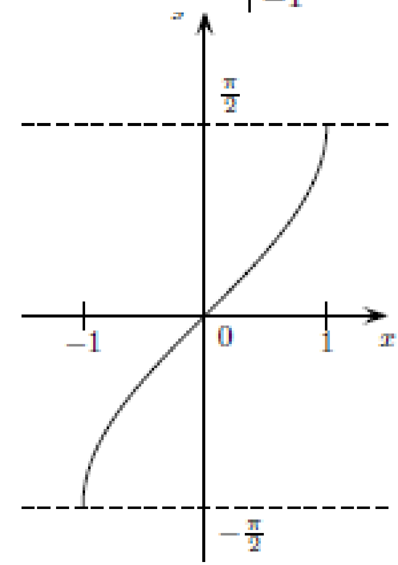
$$f(x) = \sin x$$



- L'inversa è

$$f^{-1}\colon[-1,1]\to\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$f^{-1}(x) = \arcsin(x)$$

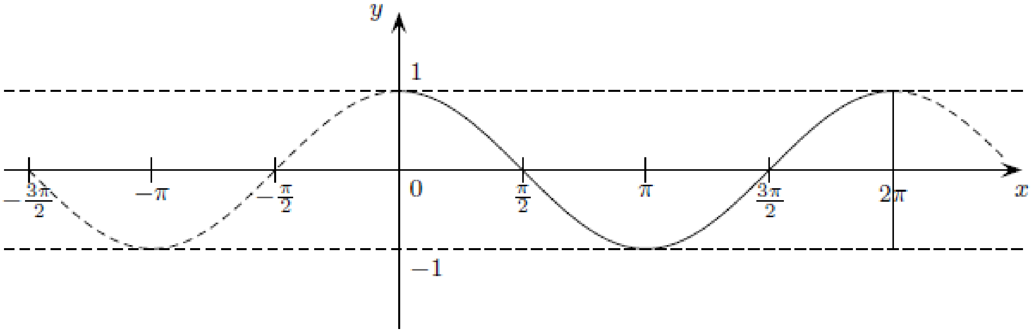


- f^{-1} è dispari

Es: $\arcsin(-1) = -\frac{\pi}{2}$; $\arcsin(0) = 0$; $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Funzioni inverse: $f^{-1}(x) = \arccos x$

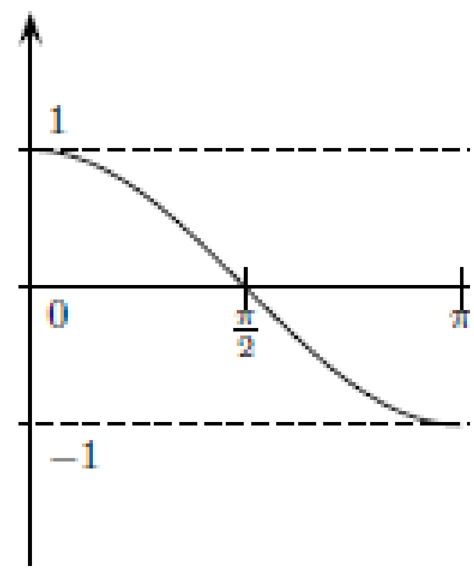
- $f: R \rightarrow R$ definita da $f(x) = \cos x$



NON è invertibile

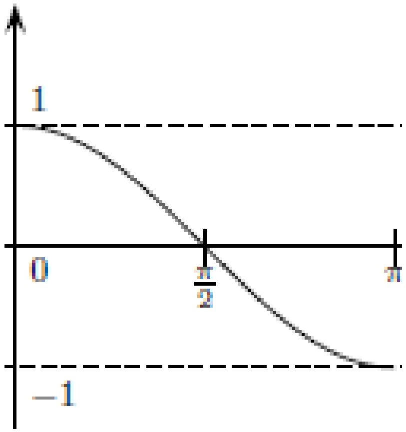
- E' invertibile se definisco

$$f: [0, \pi\ ] \rightarrow [-1,1]$$
$$f(x) = \cos x$$

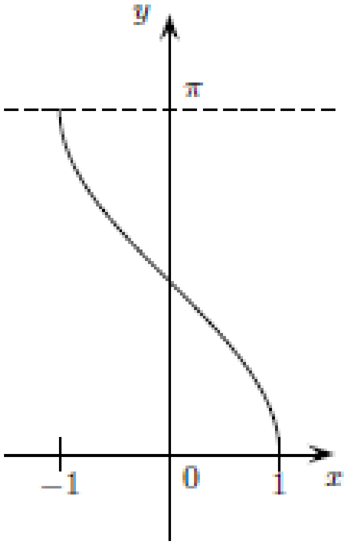


La funzione arccos(x)

$$f: [0, \pi\ ] \rightarrow [-1,1]$$
$$f(x) = \cos x$$



- L'inversa è
  $f^{-1}: [-1,1] \rightarrow [0, \pi]$
  $f^{-1}(x) = \arccos(x)$
- f^{-1} è dispari



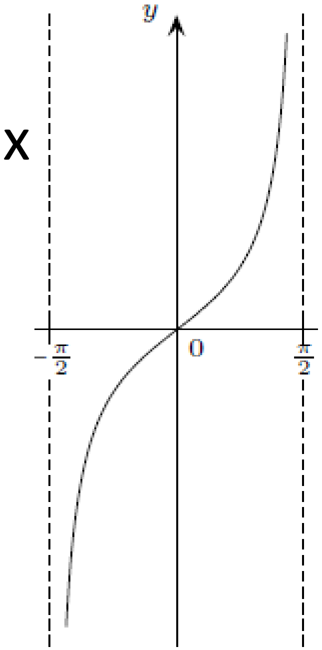
Es: arccos(-1)=π;   arccos(0)=$\frac{\pi}{2}$; $\text{arcos}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

La funzione $\arctan(x)$

- $f: (-\frac{\pi}{2}, \frac{\pi}{2}) \to R$

  $f(x) = \tan x$

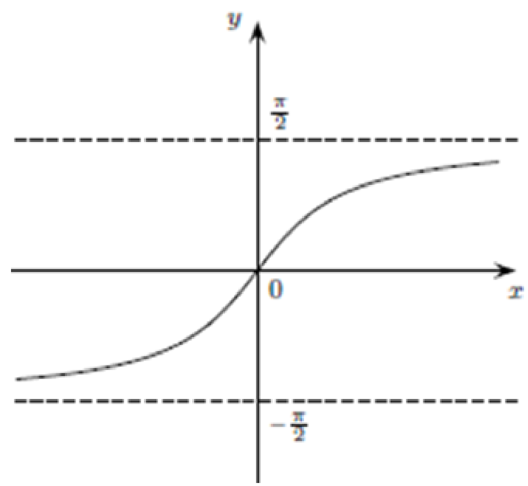
f(x)=tan x



- L'inversa è

  $f^{-1}: R \to (-\frac{\pi}{2}, \frac{\pi}{2})$

  $f^{-1}(x) = \arctan x$



- f^{-1} è dispari

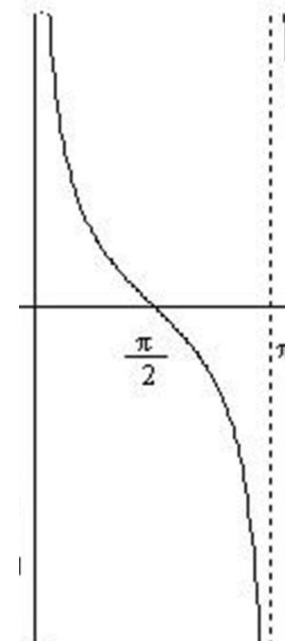
- Es: arctan(0) = 0 $\quad \arctan\sqrt{3} = \frac{\pi}{3} \quad \arctan(1) = \frac{\pi}{4}$

La funzione arccot(x)

- $f: (0, \pi) \rightarrow R$

  $f(x) = \cot x$

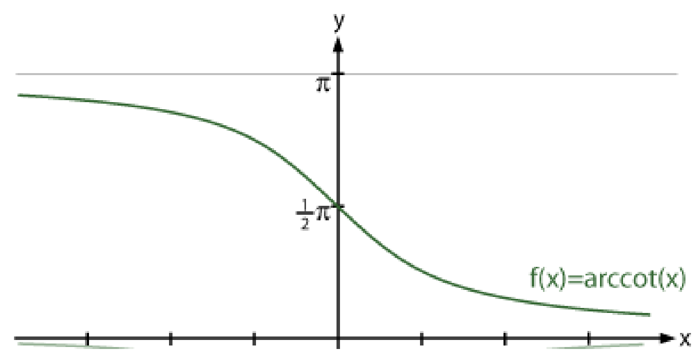
f(x)=cot x



- L'inversa è

  $f^{-1}: R \rightarrow (0, \pi)$

  $f^{-1}(x) = \text{arcco}t(x)$



Es: $\text{arccot}\, 1 = \frac{\pi}{4}$

Equazioni

- $\sin x = -2$ x=?

  $\nexists x \in R$

- $\sin x = \frac{1}{2}$ x=?

  $x = \pi - \frac{\pi}{6} + 2k\pi \qquad x = \frac{\pi}{6} + 2k\pi \quad (k \in Z)$

- $\sin 2x = \frac{\sqrt{2}}{2}$ x=?

  $2x = \frac{\pi}{4} + 2k\pi \qquad 2x = \pi - \frac{\pi}{4} + 2k\pi$

  quindi x=.....

Equazioni

- sin 5x = sin 3x

  5x=3x+2kπ 5x= π $-$3x+2kπ

  quindi x=$k\pi$ e $x = \frac{\pi}{8} + \frac{k\pi}{4}$

- $sin^2x + \sin x$ $-$2=0

Posto $\sin x = t$, si ha $t^2 + t - 2 = 0$

con soluzioni $t = -2,\ t = 1$ cioè

$\sin x = -2$ e $\sin x = 1$, ovvero

x= $\frac{\pi}{2} + 2k\pi.$

Equazioni

- $\cos x = \frac{\sqrt{3}}{2}$ x=?

$x = \frac{\pi}{6} + 2k\pi \qquad x = -\frac{\pi}{6} + 2k\pi \quad (k \in Z)$

- $\cos 7x = \cos \frac{x}{3}$ x=?

$7x = \frac{x}{3} + 2k\pi \qquad 7x = -\frac{x}{3} + 2k\pi$

- $\sqrt{2} \sin x \cos x + \cos x = 0$

$\cos x \left(\sqrt{2} \sin x + 1\right) = 0$

$x = \frac{\pi}{2} + k\pi \quad x = \frac{5\pi}{4} + 2k\pi \quad x = \frac{7\pi}{4} + 2k\pi$

Equazioni

- $\tan 4x = -1$

$$4x = \frac{3}{4}\pi + k\pi \;\rightarrow x = \frac{3}{16}\pi + k\frac{\pi}{4}$$

- $\sin^2 x \; - 3\cos^2 x = 0$

Usando la Formula fondamentale della trigonometria si ha

$$1 - \cos^2 x \; - 3\cos^2 x = 0$$

$$x = \pm\frac{\pi}{3} + k\pi$$

Disequazioni

- $\sin x > -\frac{1}{2}$

  Si risolve prima $\sin x = -\frac{1}{2}$

  $x = \frac{7\pi}{6} + 2k\pi \qquad x = -\frac{\pi}{6} + 2k\pi$

- Soluzione:

  $-\frac{\pi}{6} + 2k\pi < x < \frac{7\pi}{6} + 2k\pi$

Disequazioni

- $-\cos x - \frac{\sqrt{2}}{2} \geq 0$

Si risolve prima $\cos x + \frac{\sqrt{2}}{2} = 0$

$x = \frac{3\pi}{4} + 2k\pi \qquad x = \frac{5\pi}{4} + 2k\pi$

- Soluzione:

$\frac{3\pi}{4} + 2k\pi \leq x \leq \frac{5\pi}{4} + 2k\pi$

Disequazioni

- $\boldsymbol{\sqrt{3}\sin x - \cos x > 0}$

Dividiamo per $\cos x \neq 0$, $\left(x \neq \frac{\pi}{2} + k\pi\right)$ e distinguiamo:

1. $\cos x > 0$

2. $\cos x < 0$

1. $\begin{cases} \frac{\sqrt{3}\sin x - \cos x}{\cos x} > 0 \\ \cos x > 0 \end{cases}$

2. $\begin{cases} \frac{\sqrt{3}\sin x - \cos x}{\cos x} < 0 \\ \cos x < 0 \end{cases}$

cioè

1. $\begin{cases} \sqrt{3}\tan x - 1 > 0 \\ \cos x > 0 \end{cases}$

2. $\begin{cases} \sqrt{3}\tan x - 1 < 0 \\ \cos x < 0 \end{cases}$

1. Soluzione del primo sistema

$$\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$

2. Soluzione del secondo sistema

$$\frac{\pi}{2} + 2k\pi < x < \frac{7\pi}{6} + 2k\pi$$

- Poiché anche $x = \frac{\pi}{2} + 2k\pi$ è soluzione, facendo l'unione si ha:

$$\frac{\pi}{6} + 2k\pi < x < \frac{7\pi}{6} + 2k\pi$$