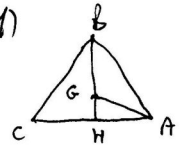
1)



$AB = a \qquad AH = \frac{a}{2}$

$BH = \frac{\sqrt{3}}{2} a \qquad GA = \frac{1}{\sqrt{3}} a$

$GH = \frac{1}{2\sqrt{3}} a$

$A^{(T)} = \frac{\sqrt{3}}{4} a^2 \qquad A^{(C)} = \pi \frac{a^2}{12} \qquad A^{(L)} = A^{(T)} - A^{(C)} = \frac{3\sqrt{3} - \pi}{12} a^2$

$m^{(T)} = \frac{A^{(T)}}{A^{(L)}} m = \frac{3\sqrt{3}}{3\sqrt{3} - \pi} m \qquad m^{(C)} = \frac{A^{(C)}}{A^{(L)}} m = \frac{\pi}{3\sqrt{3} - \pi} m$

$I^{(T)}_{H\,ij} = m^{(T)} \, \mathrm{diag}\left(\frac{a^2}{8}, \frac{a^2}{24}, \frac{a^2}{6} \right)$

$I^{(T)}_{G\,ij} = I^{(T)}_{H\,ij} - m^{(T)} \, \mathrm{diag}\left(\frac{a^2}{12}, 0, \frac{a^2}{12} \right) = m^{(T)} \left(\frac{a^2}{24}, \frac{a^2}{24}, \frac{a^2}{12} \right)$

$I^{(C)}_{G\,ij} = m^{(C)} \, \mathrm{diag}\left(\frac{a^2}{48}, \frac{a^2}{48}, \frac{a^2}{24} \right)$

$I^{(L)}_{G\,ij} = I^{(T)}_{G\,ij} - I^{(C)}_{G\,ij} = \frac{ma^2}{3\sqrt{3} - \pi} \left[\mathrm{diag}\left(\frac{\sqrt{3}}{8}, \frac{\sqrt{3}}{8}, \frac{\sqrt{3}}{4} \right) - \mathrm{diag}\left(\frac{\pi}{48}, \frac{\pi}{48}, \frac{\pi}{24} \right) \right] =$

$= \frac{6\sqrt{3} - \pi}{48(3\sqrt{3} - \pi)} ma^2 \, \mathrm{diag}(1, 1, 2)$

2) $T = \frac{1}{2} m v_G^2 + \frac{1}{2} I^{(L)}_{33} \omega^2 = \frac{1}{2} m \dot{s}^2 + \frac{1}{2} I \dot{\theta}^2 \qquad I = \frac{6\sqrt{3} - \pi}{3\sqrt{3} - \pi} \frac{ma^2}{24}$

$V = \frac{1}{2} k (G - O)^2 + mg y_G - \bar{F} \cdot \bar{x}_A = \frac{1}{2} k s^2 - mgs - \frac{aF}{\sqrt{3}} \sin\theta$

$L = T - V = \frac{1}{2} m \dot{s}^2 + \frac{1}{2} I \dot{\theta}^2 - \frac{1}{2} k s^2 + mgs + \frac{aF}{\sqrt{3}} \sin\theta$

$\frac{d}{dt} \frac{\partial L}{\partial \dot{s}} - \frac{\partial L}{\partial s} = 0 \qquad m\ddot{s} + ks - mg = 0$

$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \qquad I\ddot{\theta} - \frac{aF}{\sqrt{3}} \cos\theta = 0$

3) $\frac{\partial V}{\partial s} = ks - mg \qquad \frac{\partial V}{\partial \theta} = -\frac{aF}{\sqrt{3}} \cos\theta$

equilibrio: 1) $s = \frac{mg}{k} \quad \theta = \frac{\pi}{2}$ 2) $s = \frac{mg}{k} \quad \theta = -\frac{\pi}{2}$

$\frac{\partial^2 V}{\partial s^2} = k \qquad \frac{\partial^2 V}{\partial \theta^2} = \frac{aF}{\sqrt{3}} \sin\theta \qquad \frac{\partial^2 V}{\partial s \partial \theta} = 0$

- in 1) $V_{ss} > 0$, $V_{\theta\theta} > 0$ equilibrio stabile
- in 2) $V_{ss} > 0$, $V_{\theta\theta} < 0$ equilibrio instabile

4) $\bar{\phi}_0 + k(G - O) = 0 \quad \Rightarrow \quad \bar{\phi} = ms\,\bar{e}_2$