1) $T = T^P + T^d$

$T^P = \frac{1}{2} M v_P^2 = \frac{1}{2} M \dot{y}^2$

$T^d = \frac{1}{2} m v_C^2 + \frac{1}{2} I^C_{33} \omega^2 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \frac{mR^2}{2} \frac{\dot{x}^2}{R^2} = \frac{3}{4} m \dot{x}^2$

$V = Mg\, y_P + \frac{k}{2}(P-C)^2 - \bar{F} \cdot \bar{x}_C = Mgy + \frac{k}{2}(x^2 + y^2 - 2yR) - Fx$

$= \frac{k}{2} x^2 - Fx + \frac{k}{2} y^2 + (Mg - kR) y$

$L = T - V = \frac{1}{2} M \dot{y}^2 + \frac{3}{4} m \dot{x}^2 - \frac{k}{2} x^2 + Fx - \frac{k}{2} y^2 - (Mg - kR) y$

eq. di Lagrange: $M\ddot{y} + ky + Mg - kR = 0$

$\frac{3}{2} m \ddot{x} + kx - F = 0$

2) $\frac{\partial V}{\partial x} = kx - F = 0 \qquad x = \frac{F}{k}$

$\frac{\partial V}{\partial y} = ky + Mg - kR = 0 \qquad y = R - \frac{Mg}{k}$

$\frac{\partial^2 V}{\partial x^2} = k > 0 \qquad \frac{\partial^2 V}{\partial y^2} = k > 0 \qquad \frac{\partial^2 V}{\partial x \partial y} = 0$ equilibrio stabile

3) $A = \begin{pmatrix} \frac{3}{2} m & 0 \\ 0 & M \end{pmatrix} \qquad C = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \qquad C - \lambda A = \begin{pmatrix} k - \frac{3}{2} m\lambda & 0 \\ 0 & k - M\lambda \end{pmatrix}$

$\lambda_1 = \frac{2}{3} \frac{k}{m} \qquad \lambda_2 = \frac{k}{M}$

4) $M \bar{a}_P = M\bar{g} + k(C - P) + \bar{\phi}_P$

$\bar{\phi}_P = -kx\,\bar{e}_1 + \underbrace{[M\ddot{y} + Mg + k(y - R)]}_{=0 \text{ (eq. moto)}} \bar{e}_2 = -kx\,\bar{e}_1$

5) $x = \frac{F}{k}$ è punto di equilibrio, quindi $x(t) = \frac{F}{k}$, $\dot{x}(t) = 0$

$H = T + V$ è integrale primo $\qquad H(0) = -\frac{F^2}{2k} + MgR - \frac{kR^2}{2}$

$H(\dot{y} = 0) = -\frac{F^2}{2k} + \frac{k}{2} y^2 + (Mg - kR) y = H(0)$

$\frac{k}{2} y^2 + (Mg - kR) y - (MgR - \frac{k}{2} R^2) = 0$

$y = R, \quad R - \frac{2Mg}{k} \qquad y_{min} = R - \frac{2Mg}{k}$