1) $M^{ABCD} = 2m \qquad M^{HKLM} = m \qquad \ell^{ABCD} = a \qquad \ell^{HKLM} = \frac{a}{\sqrt{2}}$

$$\underline{\underline{I}}_G^{ABCD} = \mathrm{diag}\left(\frac{ma^2}{6}, \frac{ma^2}{6}, \frac{ma^2}{3}\right) \qquad \underline{\underline{I}}_G^{HKLM} = \mathrm{diag}\left(\frac{ma^2}{24}, \frac{ma^2}{24}, \frac{ma^2}{12}\right)$$

$$\underline{\underline{I}}_G^{lam} = \underline{\underline{I}}_G^{ABCD} - \underline{\underline{I}}_G^{HKLM} = \mathrm{diag}\left(\frac{ma^2}{8}, \frac{ma^2}{8}, \frac{ma^2}{4}\right)$$

2) $\bar{\omega} = \dot{\vartheta}\,\bar{k}_3 \qquad T = \frac{1}{2} m v_G^2 + \frac{1}{2}\bar{\omega}\cdot\underline{\underline{I}}_G(\bar{\omega}) = \frac{1}{2} m\dot{s}^2 + \frac{1}{2}\frac{ma^2}{4}\dot{\vartheta}^2$

$$V = \frac{k}{2}(Q-A)^2 - \bar{F}\cdot\bar{x}_C = \frac{k}{2}(Q-A)^2 - \frac{ka}{2} y_C$$

$$A-Q = \left(s + \frac{a}{\sqrt{2}}\cos\vartheta\right)\bar{e}_1 + \left(\frac{a}{\sqrt{2}}\operatorname{sen}\vartheta - a\right)\bar{e}_2$$

$$(A-Q)^2 = s^2 + \sqrt{2}\,sa\cos\vartheta - \sqrt{2}a^2\operatorname{sen}\vartheta + \frac{3}{2}a^2$$

$$C-O = \left(s - \frac{a}{\sqrt{2}}\cos\vartheta\right)\bar{e}_1 - \frac{a}{\sqrt{2}}\operatorname{sen}\vartheta\,\bar{e}_2$$

$$V = \frac{k}{2}(s^2 + \sqrt{2}\,sa\cos\vartheta - \sqrt{2}a^2\operatorname{sen}\vartheta) + \frac{ka^2}{2\sqrt{2}}\operatorname{sen}\vartheta = \frac{k}{2}\left(s^2 + \sqrt{2}\,as\cos\vartheta - \frac{1}{\sqrt{2}}a^2\operatorname{sen}\vartheta\right)$$

$$L = T - V = \frac{1}{2}m\dot{s}^2 + \frac{1}{8}ma^2\dot{\vartheta}^2 - \frac{k}{2}\left(s^2 + \sqrt{2}\,as\cos\vartheta - \frac{a^2}{\sqrt{2}}\operatorname{sen}\vartheta\right)$$

eq. di Lagrange: $m\ddot{s} + ks + \frac{ka}{\sqrt{2}}\cos\vartheta = 0$

$$\frac{ma^2}{4}\ddot{\vartheta} - \frac{kas}{\sqrt{2}}\operatorname{sen}\vartheta - \frac{ka^2}{2\sqrt{2}}\cos\vartheta = 0$$

3) $\frac{\partial V}{\partial s} = k\left(s + \frac{a}{\sqrt{2}}\cos\vartheta\right) = 0 \quad \rightarrow \quad s = -\frac{a}{\sqrt{2}}\cos\vartheta$

$$\frac{\partial V}{\partial\vartheta} = \frac{ka}{2}\left(-\sqrt{2}\,s\operatorname{sen}\vartheta - \frac{a}{\sqrt{2}}\cos\vartheta\right) = \frac{ka^2}{2}\cos\vartheta\left(\operatorname{sen}\vartheta - \frac{1}{\sqrt{2}}\right) = 0 \quad \rightarrow \quad \begin{array}{l}\cos\vartheta = 0\\ \operatorname{sen}\vartheta = \frac{1}{\sqrt{2}}\end{array}$$

Ia) $s = 0 \quad \vartheta = \frac{\pi}{2}$ Ib) $s = 0 \quad \vartheta = \frac{3\pi}{2}$ IIa) $s = -\frac{a}{2}, \vartheta = \frac{\pi}{4}$ IIb) $s = \frac{a}{2}, \vartheta = \frac{3\pi}{4}$

$$\frac{\partial^2 V}{\partial s^2} = k > 0 \qquad \frac{\partial^2 V}{\partial\vartheta^2} = \frac{ka}{2}\left[-\sqrt{2}\,s\cos\vartheta + \frac{a}{\sqrt{2}}\operatorname{sen}\vartheta\right] = \frac{ka^2}{2}\left[\cos^2\vartheta + \frac{1}{\sqrt{2}}\operatorname{sen}\vartheta\right]$$

$$\frac{\partial^2 V}{\partial\vartheta\partial s} = -\frac{ak}{\sqrt{2}}\operatorname{sen}\vartheta \qquad \det H = \frac{a^2k^2}{2}\left[\cos^2\vartheta + \frac{1}{\sqrt{2}}\operatorname{sen}\vartheta - \operatorname{sen}^2\vartheta\right]$$

$\det H > 0$ nei punti II (stabili) $\qquad \det H < 0$ nei punti I (instabili)

4) $A = \begin{pmatrix} m & 0 \\ 0 & \frac{ma^2}{4}\end{pmatrix} \qquad C = \begin{pmatrix} k & -\frac{ka}{2} \\ -\frac{ka}{2} & \frac{ka^2}{2}\end{pmatrix} \qquad C - \lambda A = \begin{pmatrix} k - \lambda m & -\frac{ak}{2} \\ -\frac{ak}{2} & \frac{a^2}{4}(2k - \lambda m)\end{pmatrix}$

$$\det(C - \lambda A) = 0 \quad \rightarrow \quad \lambda^2 m^2 - 3km\lambda + k^2 = 0 \qquad \lambda = \frac{3 \pm \sqrt{5}}{2}\frac{k}{m}$$

5) $\bar{L}_O = m(\bar{x}_G - \bar{x}_O)\wedge\bar{v}_G + \underline{\underline{I}}_O(\bar{\omega}) = m\,\cancel{s\bar{e}_1 \wedge \dot{s}\bar{e}_1} + \frac{ma^2}{4}\dot{\vartheta}\,\bar{e}_3$