1) $\underset{G}{I}_{ij}^{(an)} = \mathrm{diag}\left(\frac{mR^2}{2}, \frac{mR^2}{2}, mR^2\right)$

$\underset{G}{I}_{ij}^{(AB)} = \mathrm{diag}\left(\frac{mR^2}{6}, 0, \frac{mR^2}{6}\right)$

$\underset{G}{I}_{ij}^{(CD)} = \mathrm{diag}\left(0, \frac{mR^2}{6}, \frac{mR^2}{6}\right)$

$\underset{G}{I}_{ij}^{(sist.)} = \underset{G}{I}_{ij}^{(an)} + \underset{G}{I}_{ij}^{(AB)} + \underset{G}{I}_{ij}^{(CD)} = \mathrm{diag}\left(\frac{2}{3}mR^2, \frac{2}{3}mR^2, \frac{4}{3}mR^2\right)$

$\underset{A}{I}_{ij}^{(sist)} = \underset{G}{I}_{ij}^{(sist)} + \mathrm{diag}\left(2mR^2, 0, 2mR^2\right) = \mathrm{diag}\left(\frac{8}{3}mR^2, \frac{2}{3}mR^2, \frac{10}{3}mR^2\right)$

2) $T = \frac{1}{2}\omega \cdot \underset{A}{I}^{(sist)}(\omega) + \frac{1}{2}m v_P^2 = \frac{1}{2}\underset{A}{I}_{ij}^{(sist)}\omega^2 + \frac{1}{2}M v_P^2$

$\omega = \dot\vartheta \bar e_3 \qquad \bar v_P = \dot s \bar e_1 \qquad T = \frac{5}{3}mR^2\dot\vartheta^2 + \frac{1}{2}M\dot s^2$

$V = \frac{k}{2}(P-B)^2 + 2m\,y_G$

$(P-B)^2 = (s - 2R\,\mathrm{sen}\vartheta)^2 + (2R + 2R\cos\vartheta)^2 = s^2 - 4Rs\,\mathrm{sen}\vartheta + 8R^2\cos\vartheta + 4R^2$

$y_G = -(2R + R\cos\vartheta)$

$V = \frac{k}{2}(s^2 - 4Rs\,\mathrm{sen}\vartheta + 8R^2\cos\vartheta) - 2mgR\cos\vartheta + \mathrm{cost} = \frac{mg}{R}\left(\frac{s^2}{2} - 2Rs\,\mathrm{sen}\vartheta + 2R^2\cos\vartheta\right)$

$L = T - V = \frac{5}{3}mR^2\dot\vartheta^2 + \frac{1}{2}M\dot s^2 - \frac{mg}{R}\left(\frac{s^2}{2} - 2Rs\,\mathrm{sen}\vartheta + 2R^2\cos\vartheta\right)$

eq. di Lagrange:

$\frac{10}{3}mR^2\ddot\vartheta - 2mg\left(s\cos\vartheta + R\,\mathrm{sen}\vartheta\right) = 0$

$M\ddot s + \frac{mg}{R}(s - 2R\,\mathrm{sen}\vartheta) = 0$

3) $\frac{\partial V}{\partial s} = s - 2R\,\mathrm{sen}\vartheta = 0 \qquad s = 2R\,\mathrm{sen}\vartheta$

$\frac{\partial V}{\partial \vartheta} = -2Rs\cos\vartheta - 2R^2\,\mathrm{sen}\vartheta = 0 \qquad \mathrm{sen}\vartheta\,(2\cos\vartheta + 1) = 0$

Soluzioni: $\mathrm{sen}\vartheta = 0$, $\cos\vartheta = -\frac{1}{2}$

I) $\vartheta = 0$, $s = 0$; II) $\vartheta = \pi$, $s = 0$; III) $\vartheta = \frac{2\pi}{3}$, $s = -\sqrt3 R$; IV) $\vartheta = \frac{4\pi}{3}$, $s = \sqrt3 R$.

$\frac{\partial^2 V}{\partial s^2} = 1 > 0, \frac{\partial^2 V}{\partial s\partial\vartheta} = -2R\cos\vartheta, \qquad \frac{\partial^2 V}{\partial\vartheta^2} = 2R(s\,\mathrm{sen}\vartheta - R\cos\vartheta) = 2R^2(2\,\mathrm{sen}^2\vartheta - \cos\vartheta)$.

all'equilibrio

$\det \mathcal{H} = 4R^2\,\mathrm{sen}^2\vartheta - 2R^2\cos\vartheta - 4R^2\cos^2\vartheta = 2R^2(2 - 4\cos^2\vartheta - \cos\vartheta)$

$\det\mathcal{H} < 0$ nei punti I e II (instabili) ; $\det\mathcal{H} > 0$ nei punti III e IV (stabili)

4) $2m\bar a_G = 2m\bar g - k(B-P) + \bar\phi_A$