1) $\underset{C}{I}^{(d)} = \mathrm{diag}\left(\frac{ml^2}{16}, \frac{ml^2}{16}, \frac{ml^2}{8}\right)$

$\underset{C}{I}^{(a)} = \mathrm{diag}\left(\frac{16}{3}ml^2, 0, \frac{16}{3}ml^2\right)$

$\underset{C}{I}^{tot} = \underset{C}{I}^{(d)} + \underset{C}{I}^{(a)} = \left(\frac{259}{48}ml^2, \frac{ml^2}{16}, \frac{131}{24}ml^2\right)$ $\qquad \bar d = C - G = -l\bar e_2$

$\underset{G}{I}^{tot} = \underset{C}{I}^{tot} - 2m(l^2, 0, l^2) = ml^2\left(\frac{163}{48}, \frac{1}{16}, \frac{83}{24}\right)$

2) $\bar x_G = (s - l\cos\theta)\bar e_1 + \left(\frac{l}{2} - l\,\mathrm{sen}\,\theta\right)\bar e_2$

$\bar v_G = (\dot s + l\,\mathrm{sen}\,\theta\,\dot\theta)\bar e_1 - l\cos\theta\,\dot\theta\,\bar e_2$ $\qquad v_G^2 = \dot s^2 + 2l\,\mathrm{sen}\,\theta\,\dot s\dot\theta + l^2\dot\theta^2$

$\bar\omega = \dot\theta\,\bar e_3$

$T = \frac{1}{2}2mv_G^2 + \frac{1}{2}\underset{G}{I}_{33}\dot\theta^2 = m(\dot s^2 + 2l\,\mathrm{sen}\,\theta\,\dot s\dot\theta + l^2\dot\theta^2) + \frac{83}{48}\dot\theta^2 ml^2$

$= m\left(\dot s^2 + 2l\,\mathrm{sen}\,\theta\,\dot s\dot\theta + \frac{131}{48}l^2\dot\theta^2\right)$

$B - A = (s - 4l\cos\theta)\bar e_1 - \left(\frac{l}{2} + 4l\,\mathrm{sen}\,\theta\right)\bar e_2$ $\qquad (B-A)^2 = s^2 - 8ls\cos\theta + 4l^2\,\mathrm{sen}\,\theta + \frac{17}{4}l^2$

$V = \frac{k}{2}(B-A)^2 + 2mg\,y_G = \frac{mg}{10l}(s^2 - 8ls\cos\theta + 4l^2\,\mathrm{sen}\,\theta) - 2mgl\,\mathrm{sen}\,\theta$

$= \frac{mg}{5}\left(\frac{s^2}{2} - 4ls\cos\theta - 8l^2\,\mathrm{sen}\,\theta\right)$

$L = T - V = m\left(\dot s^2 + 2l\,\mathrm{sen}\,\theta\,\dot s\dot\theta + \frac{131}{48}l^2\dot\theta^2\right) - \frac{mg}{5}\left(\frac{s^2}{2} - 4ls\cos\theta - 8l^2\,\mathrm{sen}\,\theta\right)$

eq. di Lagrange: $2\ddot s + 2l\,\mathrm{sen}\,\theta\,\ddot\theta + 2l\cos\theta\,\dot\theta^2 + \frac{g}{5}(s - 4l\cos\theta) = 0$

$\frac{131}{24}l^2\ddot\theta + 2l\,\mathrm{sen}\,\theta\,\ddot s + \frac{g}{5}(4ls\,\mathrm{sen}\,\theta - 8l^2\cos\theta) = 0$

3) $\frac{\partial V}{\partial s} = s - 4l\cos\theta = 0 \qquad s = 4l\cos\theta$

$\frac{\partial V}{\partial\theta} = 4ls\,\mathrm{sen}\,\theta - 8l^2\cos\theta = 8l^2\cos\theta(2\,\mathrm{sen}\,\theta - 1) = 0 \qquad \mathrm{sen}\,\theta = \frac{1}{2}, \cos\theta = 0$

Ia) $\theta = \frac{\pi}{2}, s = 0$ $\qquad$ Ib) $\theta = \frac{3\pi}{2}, s = 0$ $\qquad$ IIa) $\theta = \frac{\pi}{6}, s = 2\sqrt{3}l$ $\qquad$ IIb) $\theta = \frac{5\pi}{6}, s = -2\sqrt{3}l$

$\frac{\partial^2 V}{\partial s^2} = 1 > 0 \qquad \frac{\partial^2 V}{\partial\theta^2} = 4ls\cos\theta + 8l^2\,\mathrm{sen}\,\theta \qquad \frac{\partial^2 V}{\partial s\partial\theta} = 4l\,\mathrm{sen}\,\theta$

$H = 4ls\cos\theta + 8l^2\,\mathrm{sen}\,\theta - 16l^2\,\mathrm{sen}^2\theta = 16l^2\cos^2\theta + 8l^2\,\mathrm{sen}\,\theta - 16l^2\,\mathrm{sen}^2\theta$

Nei punti I, $H < 0$ → eq. instabile; nei punti II, $H > 0$ → eq. stabile

4) $A = \begin{pmatrix} 2m & ml \\ ml & \frac{131}{24}l^2 m \end{pmatrix} \qquad C = \frac{mg}{5}\begin{pmatrix} 1 & 2l \\ 2l & 16l^2 \end{pmatrix}$

$\hat\lambda = \frac{mg}{5}\lambda \qquad A - \lambda C = m\begin{pmatrix} 2-\hat\lambda & l(1-2\hat\lambda) \\ l(1-2\hat\lambda) & l^2\left(\frac{131}{24} - 16\hat\lambda\right) \end{pmatrix}$ $\qquad \det(A - \lambda C) = 0$ dà λ.

5) $2m\bar a_G = 2m\bar g + k(A - B) + \bar\Phi_H$