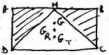

1) $m^R = 2m \qquad m^T = m$

$$HG = \frac{1}{m}\left(2m\cdot\frac{\ell}{2} - m\cdot\frac{2}{3}\ell\right) = \frac{\ell}{3}$$

$$I^R_{G_R} = m\ell^2\left(\frac{1}{6}, \frac{2}{3}, \frac{5}{6}\right) \qquad I^R_G = I^R_{G_R} + m\ell^2\left(\frac{1}{18}, 0, \frac{1}{18}\right) = m\ell^2\left(\frac{2}{9}, \frac{2}{3}, \frac{8}{9}\right)$$

$$I^T_{G_T} = m\ell^2\left(\frac{1}{18}, \frac{1}{6}, \frac{2}{9}\right) \qquad I^T_G = I^T_{G_T} + m\ell^2\left(\frac{1}{9}, 0, \frac{1}{9}\right) = m\ell^2\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{3}\right)$$

$$I^L_G = I^R_G - I^T_G = m\ell^2\left(\frac{1}{18}, \frac{1}{2}, \frac{5}{9}\right)$$

2) $\bar{x}_G = (s+\ell)\bar{e}_1 + \frac{\ell}{3}\sin\vartheta\,\bar{e}_2 - \frac{\ell}{3}\cos\vartheta\,\bar{e}_3$

$\bar{v}_G = \dot{s}\bar{e}_1 + \frac{\ell}{3}\cos\vartheta\,\dot\vartheta\,\bar{e}_2 + \frac{\ell}{3}\sin\vartheta\,\dot\vartheta\,\bar{e}_3 \qquad v_G^2 = \dot{s}^2 + \frac{\ell^2}{9}\dot\vartheta^2$

$\bar\omega = \dot\vartheta\bar{k}_1 \qquad \bar\omega\cdot I_G(\bar\omega) = I_{G11}\dot\vartheta^2$

$$T = \frac{1}{2}m v_G^2 + \frac{1}{2}I_{G11}\dot\vartheta^2 = \frac{1}{2}m\left(\dot s^2 + \frac{\ell^2}{9}\dot\vartheta^2\right) + \frac{1}{2}\frac{m\ell^2}{18}\dot\vartheta^2 = \frac{1}{2}m\left(\dot s^2 + \frac{1}{6}\ell^2\dot\vartheta^2\right)$$

$$V = \frac{k}{2}(D-O)^2 + mg z_G - \bar{F}\cdot\bar{x}_C = \frac{k}{2}(s^2+\ell^2) - \frac{mg\ell}{3}\cos\vartheta - F(x_C + y_C) =$$

$$= \frac{k}{2}s^2 - \frac{mg\ell}{3}\cos\vartheta - \frac{mg}{3}(s + \ell\sin\vartheta) + \text{cost.}$$

$$L = T - V = \frac{m}{2}\left(\dot s^2 + \frac{1}{6}\ell^2\dot\vartheta^2\right) - \frac{k}{2}s^2 + \frac{mg\ell}{3}(\cos\vartheta + \sin\vartheta + s)$$

$$m\ddot s + ks - \frac{mg}{3} = 0$$

$$\frac{1}{6}m\ell^2\ddot\vartheta + \frac{mg\ell}{3}(\sin\vartheta - \cos\vartheta) = 0$$

3) $V_s = ks - \frac{mg}{3} = 0 \qquad s = \frac{mg}{3k}$

$V_\vartheta = \frac{mg\ell}{3}(\sin\vartheta - \cos\vartheta) = 0 \qquad \operatorname{tg}\vartheta = 1 \qquad \vartheta = \frac{\pi}{4}, \frac{5\pi}{4}$

$V_{ss} = k > 0 \qquad V_{\vartheta\vartheta} = \frac{mg\ell}{3}(\cos\vartheta + \sin\vartheta) \qquad V_{s\vartheta} = 0$

$V_{\vartheta\vartheta} > 0$ per $\vartheta = \frac{\pi}{4}$ (stabile) $\qquad V_{\vartheta\vartheta} < 0$ per $\vartheta = \frac{5\pi}{4}$ (instabile)

4) $\bar\phi_A + \bar\phi_B + m\bar{g} - k(D-O) + \bar{F} = 0$

$(B-A)\wedge\bar\phi_B + (G-A)\wedge m\bar{g} - k(D-A)\wedge(D-O) + (C-A)\wedge\bar{F} = 0$

$\bar\phi_A + \bar\phi_B - mg\bar{e}_3 - k\left(\cancel{\frac{mg}{3k}\bar{e}_1} + \frac{\sqrt2}{2}\ell\,\bar{e}_2 - \frac{\sqrt2}{2}\ell\,\bar{e}_3\right) + \frac{mg}{3}(\cancel{\bar{e}_1} + \bar{e}_2) = 0$

$2\ell\,\bar{e}_1\wedge\bar\phi_B + \left(\ell\bar{e}_1 + \frac{\sqrt2}{6}\ell\bar{e}_2 - \frac{\sqrt2}{6}\ell\bar{e}_3\right)\wedge(-mg\bar{e}_3) - k\ell\frac{\sqrt2}{2}(\bar{e}_2 - \bar{e}_3)\wedge\left(s\bar{e}_1 + \frac{\sqrt2}{2}\ell\bar{e}_2 - \frac{\sqrt2}{2}\ell\bar{e}_3\right)$

$+ \left(2\ell\bar{e}_1 + \frac{\sqrt2}{2}\ell\,\bar{e}_2 - \frac{\sqrt2}{2}\ell\bar{e}_3\right)\wedge\frac{mg}{3}(\bar{e}_1 + \bar{e}_2) =$

$= 2\cancel{\ell}\,\bar{e}_1\wedge\phi_B + mg\cancel{\ell}\left(\bar{e}_2 - \cancel{\frac{\sqrt2}{6}\bar{e}_1}\right) + \frac{mg\cancel{\ell}}{3}\frac{\sqrt2}{2}(\cancel{\bar{e}_3} + \cancel{\bar{e}_2}) + \frac{mg\cancel{\ell}}{3}\left[2\bar{e}_3 + \frac{\sqrt2}{2}(-\cancel{\bar{e}_3} - \cancel{\bar{e}_2} + \cancel{\bar{e}_1})\right] = 0$

$\phi_{B2} = -\frac{mg}{3} \qquad \phi_{B3} = \frac{mg}{2} \qquad \phi_{A2} = -\frac{mg}{3} + \frac{\sqrt2}{2}k\ell \qquad \phi_{A3} = \frac{mg}{2} - \frac{\sqrt2}{2}k\ell$