1) $\bar{x}_G = \left(\frac{1}{2} g t^2 + R \operatorname{sen}\vartheta\right)\bar{e}_1 - R\cos\vartheta\, \bar{e}_2$ $\qquad \bar{\omega} = \dot{\varphi}\, \bar{e}_3$

$\bar{v}_G = (gt + R\cos\vartheta\,\dot{\vartheta})\bar{e}_1 + R\operatorname{sen}\vartheta\,\dot{\vartheta}\,\bar{e}_2$

$\bar{a}_G = (g + R\cos\vartheta\,\ddot{\vartheta} - R\operatorname{sen}\vartheta\,\dot{\vartheta}^2)\,\bar{e}_1 + R(\operatorname{sen}\vartheta\,\ddot{\vartheta} + \cos\vartheta\,\dot{\vartheta}^2)\,\bar{e}_2$

$T = \frac{1}{2} m v_G^2 + \frac{1}{2}\bar{\omega}\cdot \underset{G}{I}(\bar{\omega}) = \frac{1}{2} m\left[R^2\dot{\vartheta}^2 + g^2t^2 + 2Rgt\cos\vartheta\,\dot{\vartheta}\right] + \frac{1}{2}\frac{m\ell^2}{3}\dot{\varphi}^2$

$V = mg\,y_G + \frac{K}{2}(C-D)^2 = -mgR\cos\vartheta + KR\ell\cos(\vartheta-\varphi) + \text{cost.}$

$(C-D)^2 = (R\cos\vartheta + \ell\cos\varphi)^2 + (R\operatorname{sen}\vartheta + \ell\operatorname{sen}\varphi)^2 = R^2 + \ell^2 + 2R\ell(\cos\vartheta\cos\varphi + \operatorname{sen}\vartheta\operatorname{sen}\varphi)$

$= 2R\ell(\cos\vartheta-\varphi) + R^2 + \ell^2$

$L = \frac{m}{2}(R^2\dot{\vartheta}^2 + g^2t^2 + 2Rgt\cos\vartheta\,\dot{\vartheta}) + \frac{m\ell^2}{6}\dot{\varphi}^2 + mgR\cos\vartheta - KR\ell\cos(\vartheta-\varphi)$

$= \frac{m}{2}R^2\dot{\vartheta}^2 - mgR\operatorname{sen}\vartheta + \frac{m\ell^2}{6}\dot{\varphi}^2 + mgR\cos\vartheta - KR\ell\cos(\vartheta-\varphi) + \cancel{\frac{m}{2}\frac{d}{dt}\left[\frac{g^2t^3}{3} + 2gRt\operatorname{sen}\vartheta\right]}$

$mR^2\ddot{\vartheta} = -mgR(\cos\vartheta + \operatorname{sen}\vartheta) + KR\ell\operatorname{sen}(\vartheta-\varphi)$

$\frac{m\ell^2}{3}\ddot{\varphi} = -KR\ell\operatorname{sen}(\vartheta-\varphi)$

2) $V_{eff} = mgR(\operatorname{sen}\vartheta - \cos\vartheta) + KR\ell\cos(\vartheta-\varphi)$

$V_\varphi = +KR\ell\operatorname{sen}(\vartheta-\varphi) = 0$ se $\varphi = \vartheta$, $\varphi = \vartheta + \pi$

$V_\vartheta = mgR(\cos\vartheta + \operatorname{sen}\vartheta) - KR\ell\underbrace{\operatorname{sen}(\vartheta-\varphi)}_{=0} = 0$ se $\operatorname{tg}\vartheta = -1$ $\quad \vartheta = -\frac{\pi}{4}, \frac{3\pi}{4}$

Equilibrio: 1) $\vartheta = \varphi = \frac{3\pi}{4}$ $\quad$ 2) $\vartheta = \frac{3\pi}{4}, \varphi = -\frac{\pi}{4}$ $\quad$ 3) $\vartheta = \varphi = -\frac{\pi}{4}$ $\quad$ 4) $\vartheta = -\frac{\pi}{4}, \varphi = \frac{3\pi}{4}$

$V_{\varphi\varphi} = -KR\ell\cos(\vartheta-\varphi)$ $\quad V_{\vartheta\vartheta} = -mgR(\operatorname{sen}\vartheta - \cos\vartheta) - KR\ell\cos(\vartheta-\varphi)$ $\quad V_{\vartheta\varphi} = KR\ell\cos(\vartheta-\varphi)$

~~Abbiamo~~ Nei casi 1) e 3) $V_{\varphi\varphi} < 0$, equilibrio instabile

caso 2) $\mathcal{H} = \begin{pmatrix} KR\ell & -KR\ell \\ -KR\ell & KR\ell - \sqrt{2}mgR \end{pmatrix}$, hessiano < 0, eq. instabile

caso 4) $\mathcal{H} = \begin{pmatrix} KR\ell & -KR\ell \\ -KR\ell & KR\ell + \sqrt{2}mgR \end{pmatrix}$, hessiano > 0, eq. stabile

3) $H_{eff} = \frac{m}{2}R^2\dot{\vartheta}^2 + \frac{m}{6}\ell^2\dot{\varphi}^2 + mgR(\operatorname{sen}\vartheta - \cos\vartheta) + KR\ell\operatorname{sen}(\vartheta-\varphi)$

4) $m\bar{g} + K(D-C) + \bar{\Phi} = m\bar{a}_G$ $\qquad -mg\bar{e}_2 + K(R+\ell)\bar{e}_2 + \bar{\Phi} = m\bar{a}_G(0)$

$\frac{m\ell^2}{3}\ddot{\varphi}(0) = 0$ $\qquad \cancel{m}R^{\cancel{2}}\ddot{\vartheta}(0) = -\cancel{m}g\cancel{R}$ $\qquad \bar{a}_G(0) = \bar{0}$

$\bar{\Phi} = (mg - KR - K\ell)\,\bar{e}_2$