1) $|OC| = \frac{\sqrt{3}}{2} R \qquad |CG| = \frac{\sqrt{3}}{4} R = |KG| \qquad |AB| = R$

$$\underset{G}{I}{}^{disco}_{ij} = \mathrm{diag}\left(\frac{mR^2}{8}, \frac{mR^2}{8}, \frac{mR^2}{4}\right) + \frac{m}{2}\mathrm{diag}\left(\frac{3}{16}R^2, 0, \frac{3}{16}R^2\right) = mR^2\,\mathrm{diag}\left(\frac{7}{32}, \frac{1}{8}, \frac{11}{32}\right)$$

$$\underset{G}{I}{}^{AB}_{ij} = \mathrm{diag}\left(0, \frac{mR^2}{24}, \frac{mR^2}{24}\right) + \frac{m}{2}\mathrm{diag}\left(\frac{3}{16}R^2, 0, \frac{3}{16}R^2\right) = mR^2\,\mathrm{diag}\left(\frac{3}{32}, \frac{1}{24}, \frac{13}{96}\right)$$

$$\underset{G}{I}{}^{sist} = \underset{G}{I}{}^{disco} + \underset{G}{I}{}^{AB} = \mathrm{diag}\left(\frac{5}{16}mR^2, \frac{1}{6}mR^2, \frac{23}{48}mR^2\right)$$

2) $$T = T^{OC} + T^{sist} = \frac{1}{2}\bar{\omega}^{OC}\cdot \underset{O}{I}{}^{OC}(\bar{\omega}^{OC}) + \frac{1}{2}m v_G^2 + \frac{1}{2}\bar{\omega}^{sist}\cdot \underset{G}{I}(\bar{\omega}^{sist})$$

$$\bar{\omega}^{OC} = \dot{\theta}\,\bar{e}_3 \qquad \bar{\omega}^{sist} = \dot{\varphi}\,\bar{e}_3 \qquad \underset{O}{I}{}^{OC}_{ij} = \mathrm{diag}\left(\frac{2m\ell^2}{3}, 0, \frac{2m\ell^2}{3}\right)$$

$$\bar{x}_G = \left(\ell \sin\theta + \frac{\sqrt{3}}{4}R\sin\varphi\right)\bar{e}_1 - \left(\ell\cos\theta + \frac{\sqrt{3}}{4}R\cos\varphi\right)\bar{e}_2$$

$$\bar{v}_G = \left(\ell\cos\theta\dot{\theta} + \frac{\sqrt{3}}{4}R\cos\varphi\dot{\varphi}\right)\bar{e}_1 + \left(\ell\sin\theta\dot{\theta} + \frac{\sqrt{3}}{4}R\sin\varphi\dot{\varphi}\right)\bar{e}_2$$

$$v_G^2 = \ell^2\dot{\theta}^2 + \frac{3}{16}R^2\dot{\varphi}^2 + \frac{\sqrt{3}}{2}\ell R(\cos\theta\cos\varphi + \sin\theta\sin\varphi)\dot{\theta}\dot{\varphi}$$

$$T = \frac{1}{2}\frac{2m\ell^2}{3}\dot{\theta}^2 + \frac{1}{2}\frac{23}{48}mR^2\dot{\varphi}^2 + \frac{1}{2}m\left(\ell^2\dot{\theta}^2 + \frac{3}{16}R^2\dot{\varphi}^2 + \frac{\sqrt{3}}{2}\ell R(\cos\theta\cos\varphi + \sin\theta\sin\varphi)\dot{\theta}\dot{\varphi}\right)$$

$$= \frac{5}{6}m\ell^2\dot{\theta}^2 + \frac{1}{3}mR^2\dot{\varphi}^2 + \frac{\sqrt{3}}{4}m\ell R(\cos\theta\cos\varphi + \sin\theta\sin\varphi)\dot{\theta}\dot{\varphi}$$

$$V = 2mg\,y_H + mg\,y_G - \bar{F}\cdot\bar{x}_H = -\cancel{2}mg\frac{\ell}{\cancel{2}}\cos\theta - mg\left(\ell\cos\theta + \frac{\sqrt{3}}{4}\cos\varphi\right)$$

$$-2mg\left(\ell\sin\theta + \frac{\sqrt{3}}{2}R\sin\varphi\right) =$$

$$= -2mg\ell(\cos\theta + \sin\theta) - \sqrt{3}mgR\left(\sin\varphi + \frac{1}{4}\cos\varphi\right)$$

$L = T - V$

eq. di Lagrange

$$\frac{5}{3}m\ell^2\ddot{\theta} + \frac{\sqrt{3}}{4}m\ell R(\cos\theta\cos\varphi + \sin\theta\sin\varphi)\ddot{\varphi} - \frac{\sqrt{3}}{4}m\ell R(\sin\theta\cos\varphi - \cos\theta\sin\varphi)\dot{\varphi}^2 + 2mgR(\sin\theta - \cos\theta) = 0$$

$$\frac{2}{3}mR^2\ddot{\varphi} + \frac{\sqrt{3}}{4}m\ell R(\cos\theta\cos\varphi + \sin\theta\sin\varphi)\ddot{\theta} - \frac{\sqrt{3}}{4}m\ell R(\cos\theta\sin\varphi - \sin\theta\cos\varphi)\dot{\theta}^2 + \sqrt{3}mgR\left(\frac{1}{4}\sin\varphi - \cos\varphi\right) = 0$$

3) $$\frac{\partial V}{\partial\theta} = 2mg\ell(\sin\theta - \cos\theta) = 0 \qquad \frac{\partial V}{\partial\varphi} = \sqrt{3}mgR\left(\cos\varphi - \frac{1}{4}\sin\varphi\right) = 0$$

$\mathrm{tg}\,\theta = 1 \qquad \theta = \frac{\pi}{4},\ \theta = \frac{5\pi}{4} \qquad\qquad \mathrm{tg}\,\varphi = 4 \qquad \varphi = \mathrm{arctg}\,4,\ \varphi = \mathrm{arctg}\,4 + \pi$

$$\frac{\partial^2 V}{\partial\theta^2} = 2mg\ell(\sin\theta + \cos\theta) \qquad \frac{\partial^2 V}{\partial\varphi^2} = \sqrt{3}mgR\left(\sin\varphi + \frac{1}{4}\cos\varphi\right) \qquad \frac{\partial^2 V}{\partial\varphi\partial\theta} = 0$$

all'equilibrio, $\frac{\partial^2 V}{\partial\theta^2} = 4mg\ell\cos\theta > 0$ per $\theta = \frac{\pi}{4}$ $\qquad \frac{\partial^2 V}{\partial\varphi^2} = \sqrt{3}mgR\cos\varphi\frac{17}{4} > 0$ per $\varphi = \mathrm{arctg}\,4$

L'equilibrio è stabile per $\theta = \frac{\pi}{4}$, $\varphi = \mathrm{arctg}\,4$

4) $2m\,\bar{a}_H + m\,\bar{a}_G = 3m\bar{g} + \bar{F} + \bar{\phi}_0$

5) all'equilibrio stabile, $\cos\theta_0 = \sin\theta_0 = \frac{1}{\sqrt{2}}$, $\cos\varphi_0 = \frac{1}{\sqrt{17}}$, $\sin\varphi_0 = \frac{4}{\sqrt{17}}$

$$A_{ij} = \begin{pmatrix} \frac{5}{3}m\ell^2 & \frac{5}{4}\sqrt{\frac{3}{34}}m\ell R \\ \frac{5}{4}\sqrt{\frac{3}{34}}m\ell R & \frac{2}{3}mR^2 \end{pmatrix} \qquad C_{ij} = \begin{pmatrix} 2\sqrt{2}mg\ell & 0 \\ 0 & \frac{\sqrt{51}}{4}mgR \end{pmatrix}$$

$$\det(C - \omega^2 A) = 0$$