

$$1) T_{OC} = \frac{1}{2} \bar{\omega} \cdot \mathbf{I}_O(\bar{\omega}) = \frac{1}{2} I_{O33} \dot{\theta}^2 = \frac{1}{2} \frac{8}{3} m l^2 \dot{\theta}^2$$

$$T_{AB} = \frac{1}{2} m v_C^2 + \frac{1}{2} \bar{\omega}' \cdot \mathbf{I}_C(\bar{\omega}') = \frac{1}{2} m v_C^2 + \frac{1}{2} I_{C33} \dot{\varphi}^2$$

$$\bar{x}_C = 2l \sin \theta \bar{e}_1 - 2l \cos \theta \bar{e}_2 \quad \bar{v}_C = 2l \cos \theta \dot{\theta} \bar{e}_1 + 2l \sin \theta \dot{\theta} \bar{e}_2$$

$$v_C^2 = 4l^2 \dot{\theta}^2$$

$$T_{AB} = 2m l^2 \dot{\theta}^2 + \frac{1}{2} \frac{m l^2}{12} \dot{\varphi}^2$$

$$T = T_{OC} + T_{AB} = \frac{10}{3} m l^2 \dot{\theta}^2 + \frac{1}{24} m l^2 \dot{\varphi}^2$$

$$V = 2m \gamma_G + m \gamma_C - \bar{F} \cdot \bar{x}_B = -2mgl \cos \theta - 2mgl \cos \theta - (-2mg) \left(2l \sin \theta + \frac{l \sin \varphi}{2} \right)$$

$$= -4mgl \cos \theta + 4mgl \sin \theta + mgl \sin \varphi$$

$$L = m l^2 \left(\frac{10}{3} \dot{\theta}^2 + \frac{1}{24} \dot{\varphi}^2 \right) + 4mgl (\sin \theta - \cos \theta) - \frac{1}{4} mgl \sin \varphi$$

$$\text{eq. di Lagrange: } \frac{\partial L}{\partial \theta} = 20 m l^2 \ddot{\theta} + 4mgl (\sin \theta + \cos \theta) = 0$$

$$\frac{\partial L}{\partial \varphi} = \frac{1}{12} m l^2 \ddot{\varphi} + mgl \cos \varphi = 0$$

$$2) \frac{\partial V}{\partial \theta} = 4mgl (\sin \theta + \cos \theta) = 0 \rightarrow \tan \theta = -1 \quad \theta = \frac{3}{4}\pi, \frac{7}{4}\pi$$

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$$\frac{\partial V}{\partial \varphi} = mgl \cos \varphi = 0 \rightarrow \cos \varphi = 0 \quad \varphi = \frac{\pi}{2}, \frac{3}{2}\pi$$

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$$I) \frac{3}{4}\pi, \frac{\pi}{2} \quad II) \frac{3}{4}\pi, \frac{3}{2}\pi \quad III) \frac{7}{4}\pi, \frac{\pi}{2} \quad IV) \frac{7}{4}\pi, \frac{3}{2}\pi$$

$$\frac{\partial^2 V}{\partial \theta^2} = 4mgl (\cos \theta - \sin \theta)$$

$$\frac{\partial^2 V}{\partial \varphi^2} = -mgl \sin \varphi$$

$$\frac{\partial^2 V}{\partial \theta \partial \varphi} = 0$$

$$H = \begin{pmatrix} 4mgl (\cos \theta - \sin \theta) & 0 \\ 0 & -mgl \sin \varphi \end{pmatrix}$$

$$I) V_{\varphi\varphi} < 0, H > 0 \quad II) V_{\varphi\varphi} > 0, H < 0 \quad III) V_{\varphi\varphi} < 0, H < 0 \text{ instabile}$$

$$IV) V_{\varphi\varphi} > 0, H > 0 \text{ stabile}$$

$$3) H = T + V = \frac{10}{3} m l^2 \dot{\theta}^2 + \frac{1}{24} m l^2 \dot{\varphi}^2 - 4mgl \cos \theta$$

$$p_{\varphi} = \frac{\partial L}{\partial \dot{\varphi}} = \frac{1}{12} m l^2 \dot{\varphi}$$

$$4) H(0) = 0 \quad p_{\varphi}(0) = 0$$

$$H(\theta=0, \varphi=\frac{\pi}{2}) = \frac{10}{3} m l^2 \dot{\theta}^2 + \frac{1}{24} m l^2 \dot{\varphi}^2 - 4mgl = 0$$

$$p_{\varphi}(\theta=0, \varphi=\frac{\pi}{2}) = \frac{m l^2}{12} \dot{\varphi}^2 = 0 \rightarrow \dot{\varphi} = 0 \rightarrow \dot{\theta} = \sqrt{\frac{6g}{l}}$$