1) $\underline{\underline{I}}_G^q = \text{diag}\left(\frac{me^2}{6}, \frac{me^2}{6}, \frac{me^2}{3}\right)$ $\quad |AC| = \sqrt{2}\,\ell$

$\underline{\underline{I}}_G^{BD} = \text{diag}\left(0, \frac{me^2}{6}, \frac{me^2}{6}\right)$ $\quad \underline{\underline{I}}_G^{AC} = \text{diag}\left(\frac{me^2}{6}, 0, \frac{me^2}{6}\right)$

$\underline{\underline{I}}_G^{tot} = \underline{\underline{I}}_G^q + \underline{\underline{I}}_G^{BD} + \underline{\underline{I}}_G^{AC} = \text{diag}\left(\frac{me^2}{3}, \frac{me^2}{3}, \frac{2}{3}me^2\right)$

2) $T = \frac{1}{2}4m\,\bar{v}_G^2 + \frac{1}{2}\bar{\omega}\cdot\underline{\underline{I}}_G^{tot}(\bar{\omega}) = 2m\dot{s}^2 + \frac{1}{3}me^2\dot{\theta}^2$

$V = \frac{k}{2}(0-G)^2 + 4mg\,y_G - \bar{F}\cdot\bar{x}_A = \frac{k}{2}s^2 - 4mg\frac{s}{2} - F\left(\frac{\sqrt{3}}{2}s + \frac{\ell}{\sqrt{2}}\sin\theta\right) =$

$= \frac{k}{2}s^2 - 2mgs - \frac{\sqrt{3}}{2}Fs - \frac{\ell F}{\sqrt{2}}\sin\theta$

$L = T - V = 2m\dot{s}^2 + \frac{1}{3}me^2\dot{\theta}^2 - \frac{k}{2}s^2 + 2mgs + \frac{\sqrt{3}}{2}Fs + \frac{\ell F}{\sqrt{2}}\sin\theta$

$4m\ddot{s} + ks - 2mg - \frac{\sqrt{3}}{2}F = 0$

$\frac{2}{3}me^2\ddot{\theta} - \frac{\ell F}{\sqrt{2}}\cos\theta = 0$

3) $\frac{\partial V}{\partial s} = ks - 2mg - \frac{\sqrt{3}}{2}F = 0 \quad \rightarrow s = 2mg + \frac{\sqrt{3}}{2}F = s_0$

$\frac{\partial V}{\partial \theta} = -\frac{\ell F}{\sqrt{2}}\cos\theta = 0 \quad \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{\partial^2 V}{\partial s^2} = k > 0$ $\quad \frac{\partial^2 V}{\partial \theta^2} = \frac{\ell F}{\sqrt{2}}\sin\theta$ $\quad > 0 \text{ se } \theta = \frac{\pi}{2}$ $\quad < 0 \text{ se } \theta = \frac{3\pi}{2}$ $\quad \frac{\partial^2 V}{\partial s\,\partial\theta} = 0$

$s = s_0, \theta = \frac{\pi}{2}$ eq. stabile $\quad s = s_0, \theta = \frac{3\pi}{2}$ eq. instabile

4) $A = \begin{pmatrix} 4m & 0 \\ 0 & \frac{2}{3}me^2 \end{pmatrix} \quad C = \begin{pmatrix} k & 0 \\ 0 & \frac{\ell F}{\sqrt{2}} \end{pmatrix} \quad 0 = \det(C - \lambda A) = (k - 4m\lambda)\left(\frac{\ell F}{\sqrt{2}} - \frac{2}{3}me^2\lambda\right)$

$\lambda = \omega^2 = \frac{k}{4m}, \frac{3F}{2\sqrt{2}me}$

5) $H = T + V = 2m\dot{s}^2 + \frac{1}{3}me^2\dot{\theta}^2 + \frac{k}{2}s^2 - 2mgs = \text{cost} = \frac{k}{2}s_0^2 - 2mgs_0 + \frac{1}{3}me^2\dot{\theta}_0^2$

$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{2}{3}me^2\dot{\theta} = \text{cost} \quad \Rightarrow \dot{\theta} = \text{cost} = \dot{\theta}_0$

quando $s = 0$ $\quad 2m\dot{s}^2 = \frac{k}{2}s_0^2 - 2mgs_0 \quad \rightarrow \quad \dot{s} = \sqrt{\frac{k}{4m}s_0^2 - gs_0}$