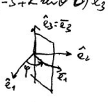
1) $T = \frac{1}{2} m v_G^2 + \bar{\omega} \cdot \underline{I}_G(\bar{\omega})$

$\bar{x}_G = R\,\mathrm{sen}\theta \cos\varphi\, \hat{e}_1 + R\,\mathrm{sen}\theta\,\mathrm{sen}\varphi\, \hat{e}_2 - (s + R\cos\theta)\hat{e}_3 \qquad \dot{\varphi} = \omega$

$\bar{v}_G = (R\cos\theta\cos\varphi\,\dot{\theta} - R\,\mathrm{sen}\theta\,\mathrm{sen}\varphi\,\dot{\varphi})\hat{e}_1 + (R\cos\theta\,\mathrm{sen}\varphi\,\dot{\theta} + R\,\mathrm{sen}\theta\cos\varphi\,\dot{\varphi})\hat{e}_2 + (-\dot{s} + R\,\mathrm{sen}\theta\,\dot{\theta})\hat{e}_3$

$v_G^2 = R^2\dot{\theta}^2 + \dot{s}^2 - 2R\,\mathrm{sen}\theta\,\dot{s}\dot{\theta} + R^2\omega^2\mathrm{sen}^2\theta$



$\bar{\omega} = \omega\bar{e}_3 + \dot{\theta}\bar{k}_3 = \omega\bar{k}_1 + \dot{\theta}\bar{k}_3$

$T = \frac{m}{2}(R^2\dot{\theta}^2 + \dot{s}^2 - 2R\,\mathrm{sen}\theta\,\dot{s}\dot{\theta} + R^2\omega^2\mathrm{sen}^2\theta) + \frac{mR^2}{\cancel{8}}\omega^2 + \frac{mR^2}{4}\dot{\theta}^2$

$= \frac{3}{4} mR^2\dot{\theta}^2 + \frac{m}{2}\dot{s}^2 - mR\,\mathrm{sen}\theta\,\dot{s}\dot{\theta} + \frac{m}{2}R^2\omega^2\mathrm{sen}^2\theta$

$V = \frac{K}{2}(A-O)^2 + mg z_G \quad = \frac{K}{2}s^2 - mgs - mgR\cos\theta$

$L = T - V = \frac{3}{4}mR^2\dot{\theta}^2 + \frac{m}{2}\dot{s}^2 - mR\,\mathrm{sen}\theta\,\dot{s}\dot{\theta} + mgR\,\mathrm{sen}^2\theta - \frac{K}{2}s^2 + mgs + mgR\cos\theta$

Eq. di Lagrange: $m\ddot{s} - mR\,\mathrm{sen}\theta\,\ddot{\theta} - mR\cos\theta\,\dot{\theta}^2 + Ks - mg = 0$

$\frac{3}{2}mR^2\ddot{\theta} - mR\,\mathrm{sen}\theta\,\ddot{s} - mgR\cos\theta\,\mathrm{sen}\theta + mgR\,\mathrm{sen}\theta = 0$

2) $\tilde{V} = V - T_0 = \frac{K}{2}s^2 - mgs - mgR\cos\theta - \frac{m}{2}R^2\omega^2\mathrm{sen}^2\theta = \frac{K}{2}s^2 - mgs - mgR(\cos\theta + \mathrm{sen}^2\theta)$

$\frac{\partial\tilde{V}}{\partial s} = Ks - mg = 0 \qquad s = \frac{mg}{K} \qquad \frac{\partial\tilde{V}}{\partial\theta} = mgR\,\mathrm{sen}\theta(1 - 2\cos\theta) = 0 \qquad \begin{cases} \mathrm{sen}\theta = 0 & \theta = 0,\pi \\ \cos\theta = \frac{1}{2} & \theta = \frac{\pi}{3}, \frac{2\pi}{3} \end{cases}$

$\frac{\partial^2\tilde{V}}{\partial s^2} = K > 0 \qquad \frac{\partial^2\tilde{V}}{\partial\theta^2} = mgR\,[\cos\theta + 2(\mathrm{sen}^2\theta - \cos^2\theta)] \qquad \frac{\partial^2 V}{\partial s\,\partial\theta} = 0$

Per $\theta = 0, \pi \qquad \frac{\partial^2\tilde{V}}{\partial\theta^2} = \pm 1 - 2 < 0$ eq. instabile

Per $\theta = \frac{\pi}{3}, \frac{2\pi}{3} \qquad \frac{\partial^2\tilde{V}}{\partial\theta^2} = \frac{1}{2} + 1 > 0$ eq. stabile

3) $s = \frac{mg}{K}$ è punto di equilibrio, quindi essendo $\dot{s}_0 = 0$, $s_0 = \frac{mg}{K}$, $s = \frac{mg}{K}$ durante tutto il moto

$H = T_2 - T_0 + V = \frac{3}{4}mR^2\dot{\theta}^2 - mgR(\cos\theta + \mathrm{sen}^2\theta) = -mgR \quad$ a $t = 0$

Per $\theta = \frac{\pi}{2} \qquad H = \frac{3}{4}mR^2\dot{\theta}^2 - mgR = -mgR \Rightarrow \dot{\theta} = 0$