

$$1) \frac{I_{ij}^{0A}}{O} = \text{diag} \left(\frac{m\ell^2}{3}, 0, \frac{m\ell^2}{2} \right)$$

$$\frac{I_{ij}^{0C}}{O} = \text{diag} \left(0, \frac{m\ell^2}{3}, \frac{m\ell^2}{3} \right)$$

$$\frac{I_{ij}^{0A+C}}{O} = \text{diag} \left(\frac{m\ell^2}{3}, \frac{m\ell^2}{3}, \frac{2m\ell^2}{3} \right)$$

$$\frac{I_{ij}^{0A+C}}{O} = \frac{I_{ij}^{0A+C}}{O} + \text{diag} (2m\ell^2, 0, 2m\ell^2)$$

$$\frac{I_{ij}^{0A+C}}{O} = \text{diag} \left(\frac{7}{3}m\ell^2, \frac{m\ell^2}{3}, \frac{8}{3}m\ell^2 \right)$$



$$2) T^{0A+C} = \frac{1}{2} \bar{\omega} \cdot \frac{I}{O}(\bar{\omega}) = \frac{1}{2} I_{33} \omega_3^2 = \frac{1}{2} m \ell^2 \dot{\theta}^2$$

$$\bar{\omega} = \dot{\theta} \bar{k}_3$$

$$T^P = \frac{1}{2} 2m v_P^2$$

$$\bar{x}_P = (\ell \sin \theta + s \cos \theta) \bar{e}_1 + (-\ell \cos \theta + s \sin \theta) \bar{e}_2$$

$$\bar{v}_P = [(\ell \cos \theta - s \sin \theta) \dot{\theta} + \dot{s} \cos \theta] \bar{e}_1 + [(-\ell \sin \theta + s \cos \theta) \dot{\theta} + \dot{s} \sin \theta] \bar{e}_2$$

$$v^2 = \dot{s}^2 + \ell^2 \dot{\theta}^2 + s^2 \dot{\theta}^2 + 2\ell \dot{s} \dot{\theta} \quad T^P = m (s^2 \dot{\theta}^2 + \ell^2 \dot{\theta}^2 + s^2 \dot{\theta}^2 + 2\ell \dot{s} \dot{\theta})$$

$$T = T^{0A+C} + T^P = m \left[\dot{s}^2 + \left(\frac{7}{3} \ell^2 + s^2 \right) \dot{\theta}^2 + 2\ell \dot{s} \dot{\theta} \right]$$

$$V = 2mg y_C + 2mg y_P + \frac{1}{2} (l - s)^2 = -2mg \ell \cos \theta - 2mg \ell \cos \theta + 2mg s \sin \theta + \frac{1}{2} (\ell^2 + s^2)$$

$$V = -4mg \ell \cos \theta + 2mg s \sin \theta + \frac{mg}{2} s^2 + \text{const}$$

$$L = T - V = m \left[\dot{s}^2 + \left(\frac{7}{3} \ell^2 + s^2 \right) \dot{\theta}^2 + 2\ell \dot{s} \dot{\theta} \right] + 4mg \ell \cos \theta - 2mg s \sin \theta - \frac{mg}{2} s^2$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{s}} - \frac{\partial L}{\partial s} = 0 \quad 2\dot{s} + 2\ell \ddot{\theta} - 2s \ddot{\theta}^2 + 2g \sin \theta + \frac{g}{2} s = 0$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \quad 2 \left(\frac{7}{3} \ell^2 + s^2 \right) \ddot{\theta} + 2\ell \dot{s} \dot{\theta} + 2s \dot{\theta} \dot{s} + 4g \ell \sin \theta + 2g s \cos \theta = 0$$

$$3) V_s = \frac{mg}{2\ell} s + 2mg \sin \theta = 0 \quad V_\theta = 4mg \ell \sin \theta + 2mg s \cos \theta = 0$$

$$s = -4\ell \sin \theta \quad 4 \sin \theta (1 - 2 \cos \theta) = 0 \quad \Rightarrow \quad \sin \theta = 0, s = 0$$

$$V_{ss} = \frac{mg}{2\ell} > 0 \quad V_{\theta\theta} = 4mg \ell \cos \theta - 2mg s \sin \theta$$

$$V_{s\theta} = 2mg \cos \theta \quad H = mg \begin{pmatrix} \frac{1}{2\ell} & 2 \cos \theta \\ 2 \cos \theta & 4\ell \cos \theta - 2s \sin \theta \end{pmatrix} = 2 \cos \theta - \frac{s}{\ell} \sin \theta - 4 \cos^2 \theta$$

$$a) \theta = 0, s = 0 \quad H < 0 \quad \text{inst.} \quad b) \theta = \pi, s = 0 \quad H < 0 \quad \text{inst.}$$

$$c) \theta = \frac{\pi}{2}, s = -2\ell \quad H > 0 \quad \text{stab.} \quad d) \theta = \frac{2\pi}{3}, s = 2\ell \quad H > 0 \quad \text{stab.}$$

$$4) \bar{L}_O = \frac{I}{O}(\bar{\omega}) + 2m \bar{x}_P \wedge \bar{v}_P = \frac{8}{3} m \ell^2 \dot{\theta} \bar{e}_3 + 2m (\ell^2 + s^2) \dot{\theta} \bar{e}_3 + 2m \ell \dot{s} \bar{e}_3$$

$$= \left[\left(\frac{14}{3} m \ell^2 + 2m s^2 \right) \dot{\theta} + 2m \ell \dot{s} \right] \bar{e}_3$$