1) $\rho = \frac{m}{A^c - A^q} = \frac{m}{(\pi-1)R^2}$ $\quad M^c = \frac{\pi}{\pi-1} m \quad M^q = \frac{1}{\pi-1} m$

$$I^d = I^c - I^q = \frac{m}{\pi-1}\left[\mathrm{diag}\left(\frac{\pi}{4}R^2, \frac{\pi}{4}R^2, \frac{\pi}{2}R^2\right) - \mathrm{diag}\left(\frac{1}{12}R^2, \frac{1}{12}R^2, \frac{1}{6}R^2\right)\right] =$$

$$= \frac{3\pi-1}{12(\pi-1)} mR^2 \ \mathrm{diag}(1,1,2)$$

2) $\bar{v}_B = \dot{s}\bar{e}_2 \qquad \bar{v}_C = 0 = \bar{v}_B + \bar{\omega}_d \wedge (C-B) = \dot{s}\bar{e}_2 + \omega_d \bar{e}_3 \wedge (-R\bar{e}_1) \Rightarrow \bar{\omega}_d = \frac{\dot{s}}{R}\bar{e}_3$

$$T^d = \frac{1}{2} m v_B^2 + \frac{1}{2} I^B_{33} \omega_d^2 = \frac{1}{2} m\dot{s}^2 + \frac{3\pi-1}{12(\pi-1)} m\dot{s}^2 = \frac{9\pi-7}{12(\pi-1)} m\dot{s}^2$$

$\bar{x}_Q = \left(R + \frac{1}{2}\sqrt{16R^2 - s^2}\right)\bar{e}_1 + \frac{s}{2}\bar{e}_2 \qquad \bar{v}_Q = \frac{-s\dot{s}}{2\sqrt{16R^2-s^2}}\bar{e}_1 + \frac{\dot{s}}{2}\bar{e}_2 \qquad v_Q^2 = \frac{4R^2\dot{s}^2}{16R^2-s^2}$

$\theta = \arcsin\left(-\frac{s}{4R}\right) \qquad \dot{\theta} = -\frac{\dot{s}}{4R}\frac{1}{\sqrt{1-\frac{s^2}{16R^2}}} = \frac{-\dot{s}}{\sqrt{16R^2-s^2}} \qquad \bar{\omega}_a = \dot{\theta}\bar{e}_3 = \frac{-\dot{s}}{\sqrt{16R^2-s^2}}\bar{e}_3$

$I^a_{Q\,33} = \frac{16mR^2}{12} = \frac{4}{3}mR^2$

$$T^a = \frac{1}{2} m v_Q^2 + \frac{1}{2} I_{Q33}\dot{\theta}^2 = \frac{2mR^2\dot{s}^2}{16R^2-s^2} + \frac{2}{3}mR^2\frac{\dot{s}^2}{16R^2-s^2} = \frac{8}{3}\frac{mR^2}{16R^2-s^2}\dot{s}^2$$

$$T^{tot} = T^d + T^a = \left[\frac{9\pi-7}{12(\pi-1)} + \frac{8}{3}\frac{R^2}{16R^2-s^2}\right]m\dot{s}^2$$

$$V = \frac{k}{2}(O-B)^2 + mg\,y_B + mg\,y_Q = \frac{k}{2}(s^2+R^2) + mgs + mg\frac{s}{2} = \frac{3mg}{8R}(s^2+4Rs)$$

$L = T - V$

3) $V_s = (2s+4R)\frac{3mg}{8R} = 0 \quad$ se $\quad s = -2R$

$V_{ss} = \frac{3mg}{4R} > 0 \quad$ equilibrio stabile

4) $T(s=-2R) \approx \left(\frac{9\pi-7}{12(\pi-1)} + \frac{2}{9}\right)m\dot{s}^2 = \frac{3\pi-29}{36(\pi-1)} m\dot{s}^2$

$V(s=-2R) \sim \frac{1}{2}V_{ss}\bar{s}^2 = \frac{3mg}{8R}\bar{s}^2 \qquad \bar{s} = s+2R$

$$\omega^2 = \frac{3mg}{4R}\cdot\frac{18(\pi-1)}{(3\pi-29)m} = \frac{27(\pi-1)}{2(3\pi-29)}\frac{g}{R}$$

5) $H = T + V = -\frac{9mgR}{8} = \frac{3\pi-29}{36(\pi-1)}m\dot{s}^2 - \frac{3mgR}{2}$

a $t=0$ $\qquad$ quando $s=-2R$

Dunque, $\dot{s}(s=-2R) = \sqrt{\frac{9(\pi-1)}{3\pi-29}\frac{3}{2}gR}$