1) $\omega = \dot\theta \mathbf{k}_3 \qquad \mathbf{x}_G = \frac{l}{2}(3\cos\theta\mathbf{e}_1 - \operatorname{sen}\theta\mathbf{e}_2)$

$\mathbf{v}_G = -\frac{l}{2}(3\operatorname{sen}\theta\mathbf{e}_1 + \cos\theta\mathbf{e}_2)\dot\theta \qquad v_G^2 = l^2(\frac{1}{4} + 2\operatorname{sen}^2\theta)\dot\theta^2$

$T_P = \frac{1}{2}M\dot s^2 \qquad T_{OA} = \frac{1}{2}I_{O_{33}}\omega_3^2 = \frac{1}{6}ml^2\dot\theta^2$

$T_{AB} = \frac{1}{2}mv_G^2 + \frac{1}{2}I_{G_{33}}\omega_3^2 = \frac{1}{2}ml^2\left(\frac{1}{4} + 2\operatorname{sen}^2\theta\right)\dot\theta^2 + \frac{1}{24}ml^2\dot\theta^2 = \left(\frac{1}{6} + \operatorname{sen}^2\theta\right)ml^2\dot\theta^2$

$T = T_P + T_{OA} + T_{AB} = \frac{1}{2}M\dot s^2 + \left(\frac{1}{3} + \operatorname{sen}^2\theta\right)ml^2\dot\theta^2$

$V = -mg\mathbf{x}_H - mg\mathbf{x}_G + \frac{k}{2}(P-A)^2 = -mg\frac{l}{2}\operatorname{sen}\theta - mg\frac{l}{2}\operatorname{sen}\theta + \frac{mg}{l}(s^2 - 2ls\cos\theta + l^2) =$
$-mgl\operatorname{sen}\theta + \frac{mg}{l}s^2 - 2mgs\cos\theta$

$L = T - V = \frac{1}{2}M\dot s^2 + \left(\frac{1}{3} + \operatorname{sen}^2\theta\right)ml^2\dot\theta^2 + mgl\operatorname{sen}\theta - \frac{mg}{l}s^2 + 2mgs\cos\theta$

Equazioni di Lagrange:

$\frac{\partial L}{\partial \dot s} = M\dot s \qquad \frac{\partial L}{\partial s} = -\frac{2mg}{l}s + 2mg\cos\theta$

$M\ddot s + \frac{2mg}{l}s - 2mg\cos\theta = 0$

$\frac{\partial L}{\partial \dot\theta} = 2\left(\frac{1}{3} + \operatorname{sen}^2\theta\right)ml^2\dot\theta \qquad \frac{\partial L}{\partial \theta} = mgl\cos\theta - 2mgs\operatorname{sen}\theta + 2ml^2\cos\theta\operatorname{sen}\theta\,\dot\theta^2$

$2\left(\frac{1}{3} + \operatorname{sen}^2\theta\right)ml^2\ddot\theta + 2ml^2\cos\theta\operatorname{sen}\theta\,\dot\theta^2 - mgl\cos\theta + 2mgs\operatorname{sen}\theta = 0$

2) Equilibrio:

$V_s = 2mg\left(\frac{s}{l} - \cos\theta\right) \qquad V_\theta = mg(2s\operatorname{sen}\theta - l\cos\theta)$

$V_s = 0 \to s = l\cos\theta \qquad V_\theta = 0 \to \cos\theta(\operatorname{sen}\theta - \frac{1}{2}) = 0$

Soluzioni:

$\cos\theta = 0, s = 0$ oppure $\operatorname{sen}\theta = \frac{1}{2}, s = \pm\frac{\sqrt3}{2}l$

I) $\theta = \frac{\pi}{2}, s = 0$; II) $\theta = \frac{3\pi}{2}, s = 0$; III) $\theta = \frac{\pi}{6}, s = \frac{\sqrt3}{2}l$; IV) $\theta = \frac{5\pi}{6}, s = -\frac{\sqrt3}{2}l$.

Stabilità:

$V_{ss} = \frac{2mg}{l} \qquad V_{\theta\theta} = mg(2s\cos\theta + l\operatorname{sen}\theta) \qquad V_{s\theta} = V_{\theta s} = 2mg\operatorname{sen}\theta$

Casi I e II) $mg\begin{pmatrix} 2/l & \pm2 \\ \pm2 & \pm l\end{pmatrix}$ $V_{ss} > 0$, $H = \pm2 - 4 < 0$: instabile

Casi III e IV) $mg\begin{pmatrix} 2/l & 1 \\ 1 & 5l/4\end{pmatrix}$ $V_{ss} > 0$, $H = 3/2 > 0$: stabile

3) Piccole oscillazioni:

$A_{ij} = \begin{pmatrix} M & 0 \\ 0 & 7ml^2/6\end{pmatrix} \qquad C_{ij} = \begin{pmatrix} 2mg/l & mg \\ mg & 5mgl/4\end{pmatrix}$

$\omega^4 - \left(\frac{5}{4} + \frac{7}{3}\frac{m}{M}\right)\frac{g}{l}\omega^2 + \frac{3}{2}\frac{m}{M}\frac{g^2}{l^2} = 0$

4) Integrali primi:

$H = T + V = \frac{1}{2}M\dot s^2 + \left(\frac{1}{3} + \operatorname{sen}^2\theta\right)ml^2\dot\theta^2 - mgl\operatorname{sen}\theta + \frac{mg}{l}s^2 - 2mgs\cos\theta$

5) Reazioni vincolari all'equilibrio:

$2m\mathbf{g} + k(P-A) + \mathbf{\Phi}_O + \mathbf{\Phi}_B = 0$

$(G-O)\wedge m\mathbf{g} + (H-O)\wedge m\mathbf{g} + k(A-O)\wedge(P-A) + (B-O)\wedge\mathbf{\Phi}_B = 0$