Virtually small spectrum of a Riemannian manifold .

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For a Riemaninan manifold (M, g) we refer to the Laplace Beltrami operator on $k-$ differential forms as the $k-$Laplacian.

In this paper we will show how to associate to a Riemannian manifold (M, g) and any rest point (zero) of a generic vector field X of Morse index k an eigenvalue of and a corresponding eigenform of the $k-$Laplacian. In case X is the gradient of a Morse function $f : M \to \mathbb{R}$ we refer to these eigenvalues as the *virtually small* eigenvalues and to the collection of these eigenvalues and eigenforms as *the virtually small spectral signature* of (M, g, f). One expects that geometric and topological information about (M, g, f) is carried on by this virtually small signature.

For example one conjecture bounds of the virtually small eigenvalues on the lines of Cheeger's result about the first nonzero eigenvalue of the $0-$Laplacian, and one proposes explanations for the experimental observations of Max Wardjewski concerning the nodal lines of some "mysterious eigenfunctions" of the $0-$Laplacian on surfaces embedded in R^3. This is work in progress.