Critical points for distance functions on surfaces

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The talk is based on joint work with I. Bárány, J. Itoh, and T. Zamfirescu.

Let S be a compact Riemannian surface without boundary. Denote by ρ the intrinsic metric on S, and by ρ_x the *distance function* from x, given by $\rho_x(y) = \rho(x,y)$. A point $y \in S$ is called *critical* with respect to ρ_x (or to x), if for any direction v of S at y there exists a segment (i.e., *shortest path*) from y to x whose direction at y makes an angle $\alpha \leq \pi/2$ with v.

We show that every point on S is critical with respect to some other point of the surface, and this lower bound is sharp. Moreover, S is homeomorphic to the sphere S^2 if and only if each point in S is critical with respect to precisely one other point of S.

Assume now that S is orientable. For a generic Riemannian metric on S, the point y is critical with respect to an odd number of points in S. For any Riemannian metric on S, y is critical with respect to at most $8g - 5$ points in S, where g is the genus of S.